Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/124-4.5.4.1-a+b-sec-^m-A+B-sec+C-sec^2-

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September 5, 2023 Compiled on September 5, 2023 at 8:02pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [70]. This is test number [124].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (70)	0.00 (0)
Mathematica	100.00 (70)	0.00 (0)
Maple	100.00 (70)	0.00 (0)
Fricas	100.00 (70)	0.00 (0)
Mupad	70.00 (49)	30.00 (21)
Maxima	68.57 (48)	31.43 (22)
Giac	65.71 (46)	34.29 (24)
Sympy	4.29 (3)	95.71 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

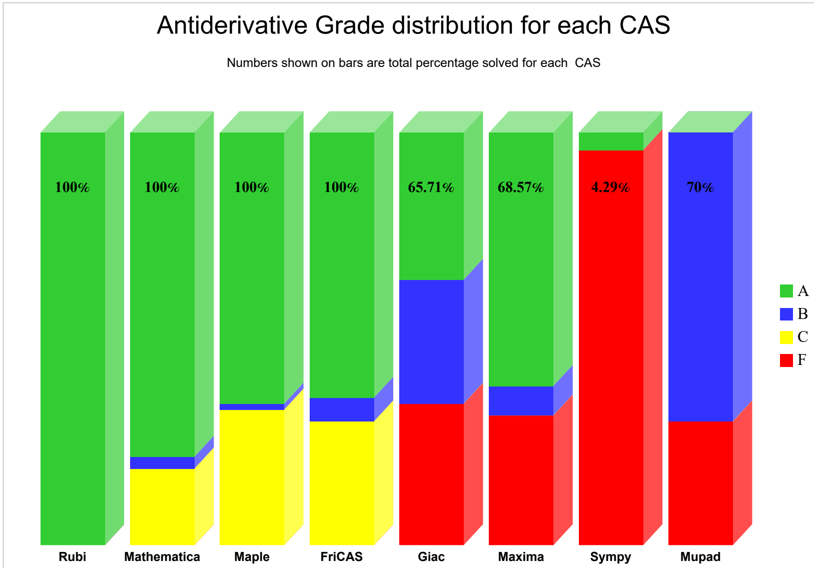
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

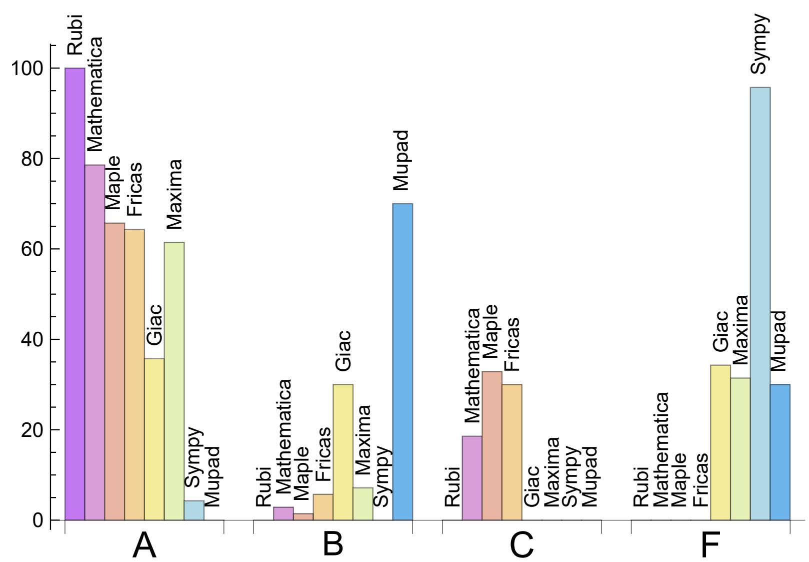
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	78.571	2.857	18.571	0.000
Maple	65.714	1.429	32.857	0.000
Fricas	64.286	5.714	30.000	0.000
Maxima	61.429	7.143	0.000	31.429
Giac	35.714	30.000	0.000	34.286
Sympy	4.286	0.000	0.000	95.714
Mupad	0.000	70.000	0.000	30.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Mupad	21	0.00	100.00	0.00
Maxima	22	100.00	0.00	0.00
Giac	24	100.00	0.00	0.00
Sympy	67	92.54	7.46	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.07
Fricas	0.21
Maxima	0.23
Giac	0.29
Mathematica	0.48
Maple	1.44
Sympy	2.07
Mupad	13.21

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	38.00	1.93	49.00	1.65
Mupad	60.37	1.37	42.00	1.10
Maxima	68.92	1.90	44.50	1.18
Rubi	69.46	1.00	64.00	1.00
Mathematica	75.79	1.27	60.50	1.00
Fricas	80.67	1.21	69.50	1.16
Giac	84.67	1.66	60.00	1.62
Maple	213.20	2.38	59.00	1.33

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

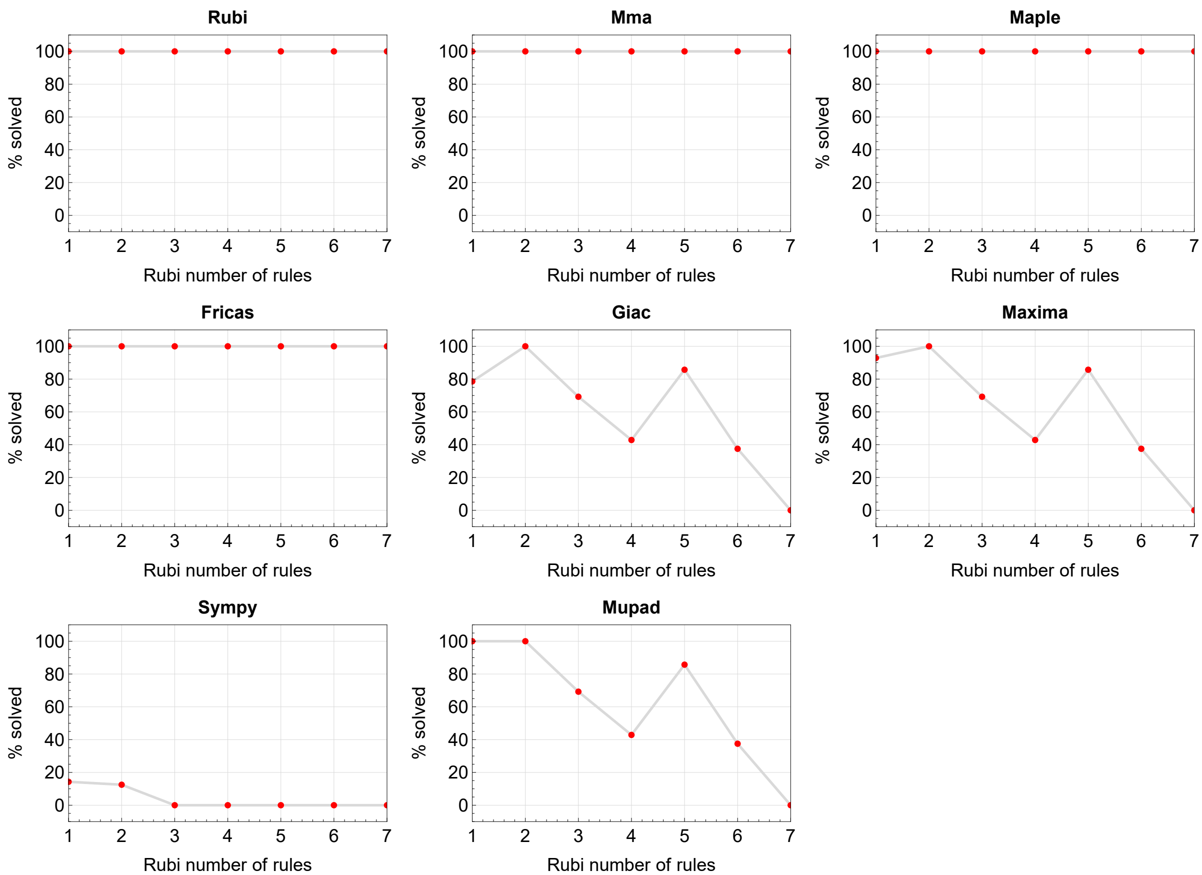


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

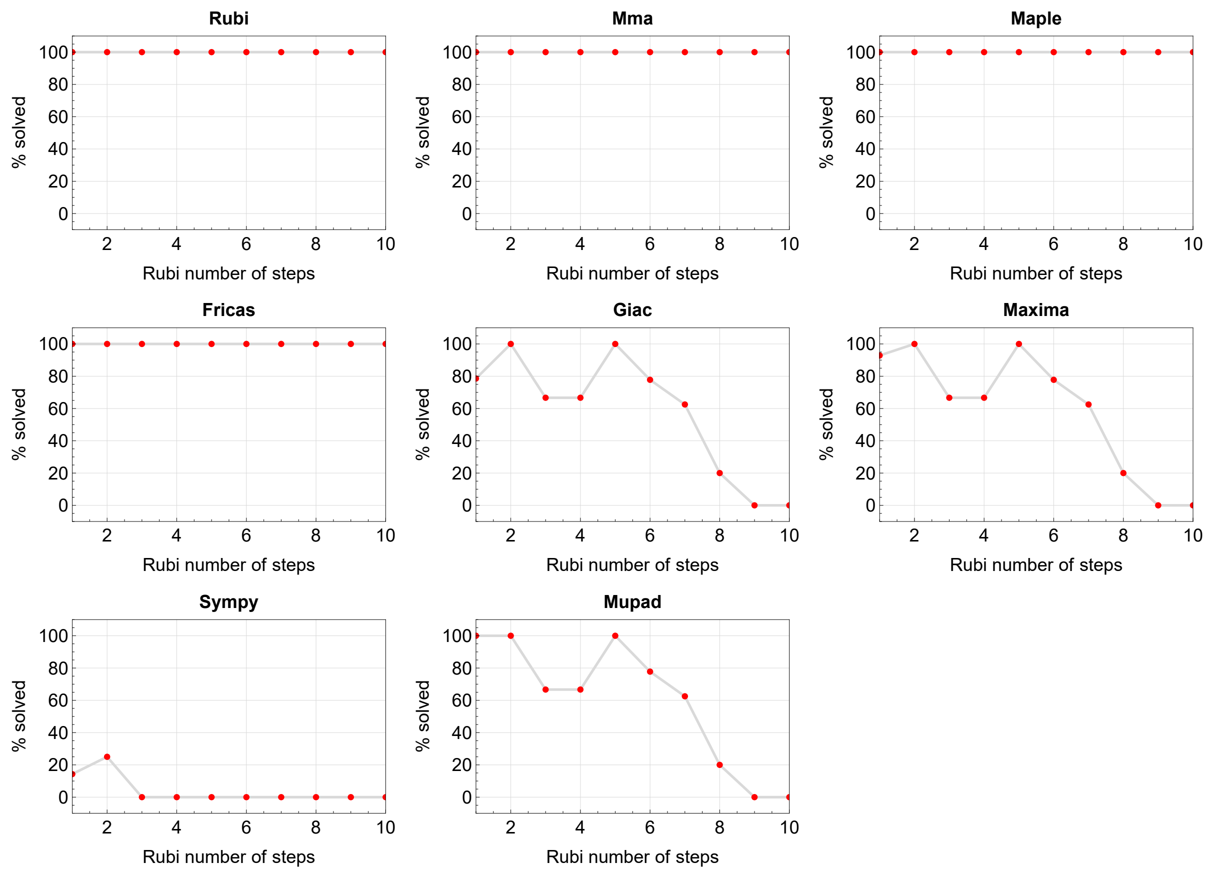


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

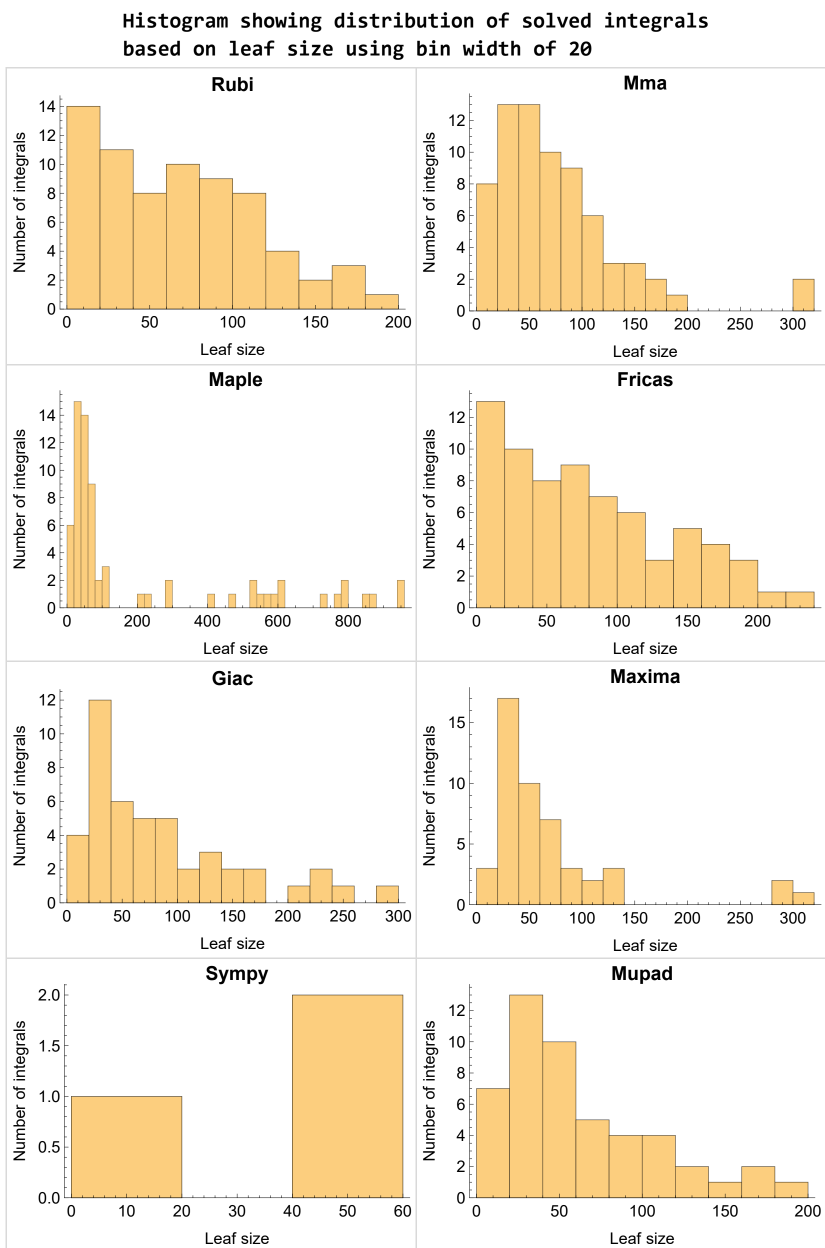


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

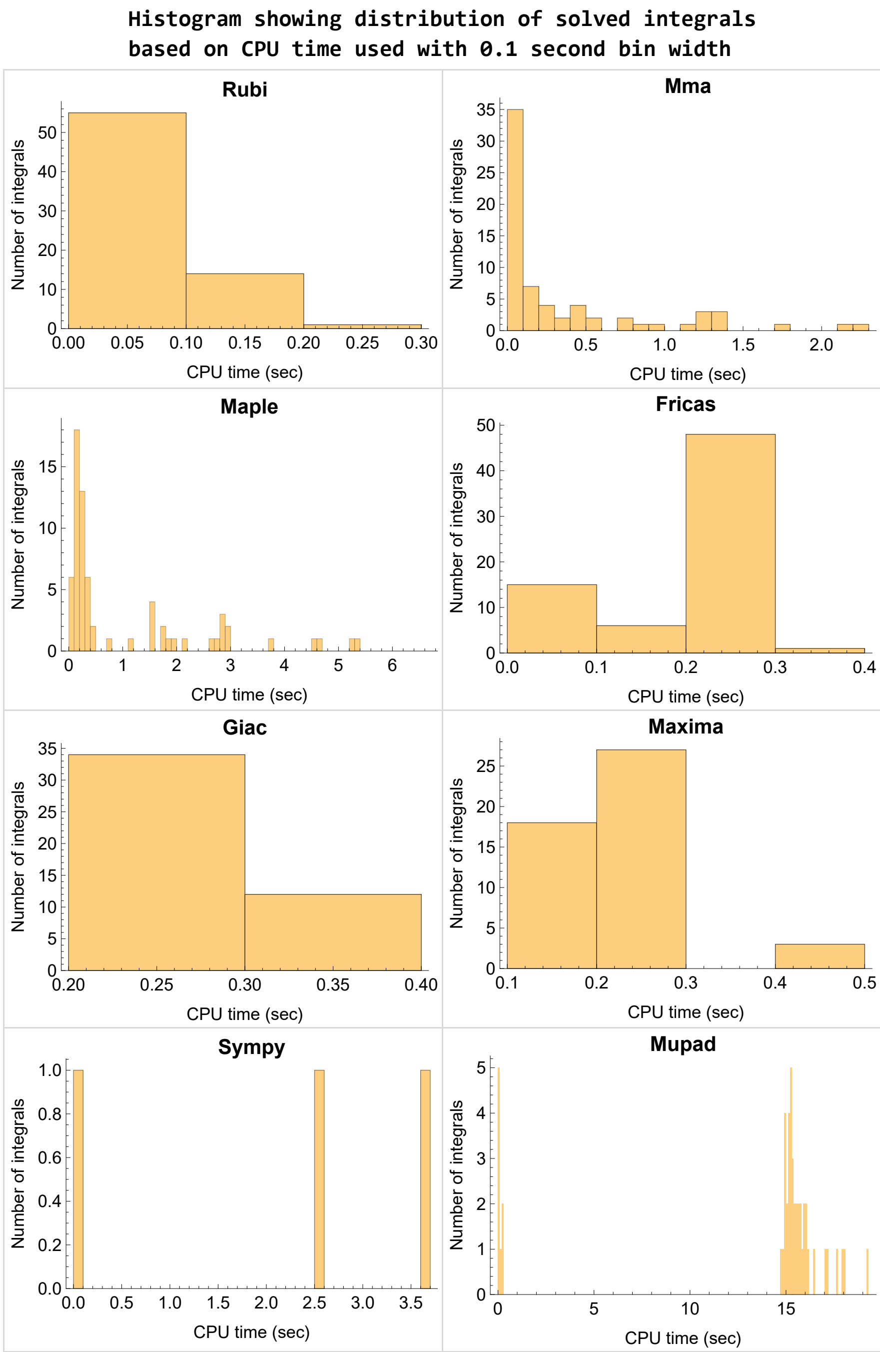


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

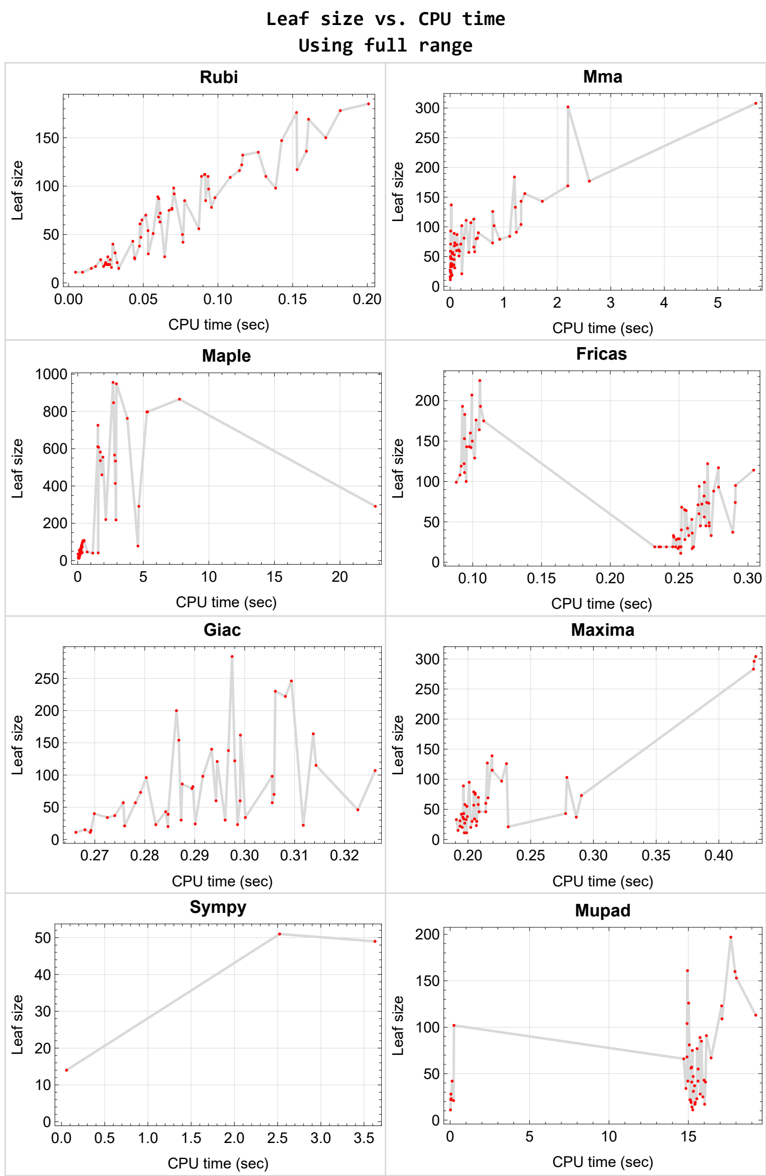


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

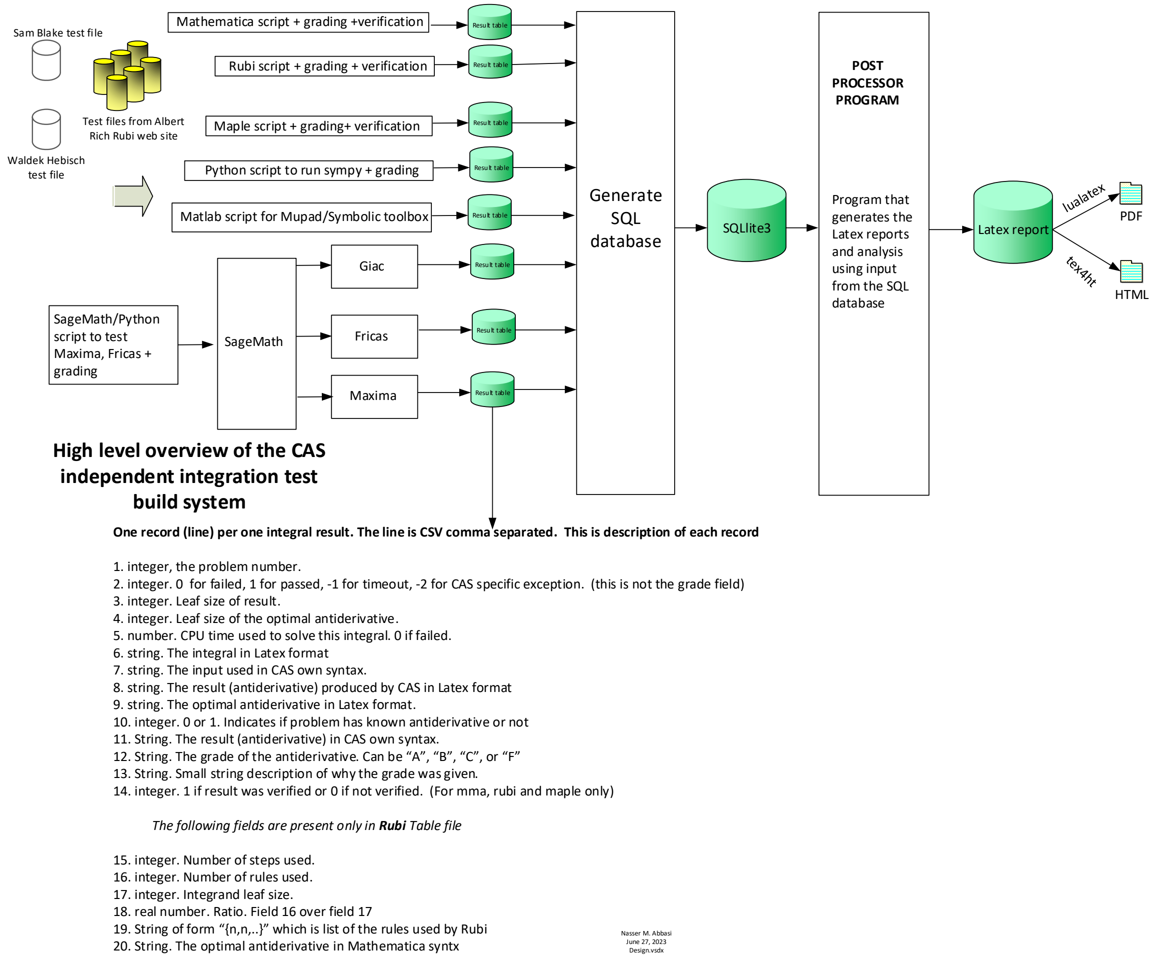
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade { 32, 34 }

C grade { 14, 15, 17, 19, 21, 23, 25, 65, 66, 67, 68, 69, 70 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 25, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade { 24 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 68, 69, 70 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64 }

B grade { 7, 40, 41, 58 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 68, 69, 70 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade { 14, 15, 25, 26, 41 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 46 }

B grade { 24, 37, 38, 39, 40, 41, 42, 43, 44, 45, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

C grade { }

F normal fail { }

F(-1) timedout fail { 16, 17, 18, 19, 20, 21, 22, 23, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 68, 69, 70 }

F(-2) exception fail { }

Sympy

A grade { 9, 32, 33 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { 13, 23, 46, 53, 64 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	60	74	0	79	56
N.S.	1	1.00	0.93	0.90	0.69	0.85	0.00	0.91	0.64
time (sec)	N/A	0.060	0.259	4.590	0.214	0.291	0.000	0.289	15.170

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	137	108	126	114	0	121	102
N.S.	1	1.00	1.40	1.10	1.29	1.16	0.00	1.23	1.04
time (sec)	N/A	0.070	0.022	0.490	0.231	0.304	0.000	0.295	0.238

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	43	56	0	57	42
N.S.	1	1.00	0.94	0.89	0.66	0.86	0.00	0.88	0.65
time (sec)	N/A	0.049	0.161	0.245	0.278	0.268	0.000	0.306	14.966

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	85	97	95	0	98	77
N.S.	1	1.00	1.33	1.21	1.39	1.36	0.00	1.40	1.10
time (sec)	N/A	0.052	0.009	0.312	0.227	0.291	0.000	0.306	15.548

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	34	37	0	34	28
N.S.	1	1.00	0.84	0.81	0.79	0.86	0.00	0.79	0.65
time (sec)	N/A	0.043	0.068	0.191	0.205	0.289	0.000	0.300	15.745

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	1.02
time (sec)	N/A	0.030	0.009	0.118	0.208	0.267	0.000	0.299	16.080

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.015	0.002	0.045	0.192	0.246	0.000	0.268	16.025

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	0.92
time (sec)	N/A	0.028	0.013	0.099	0.200	0.252	0.000	0.270	0.056

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	37	28	51	37	25
N.S.	1	1.00	1.06	0.77	1.19	0.90	1.65	1.19	0.81
time (sec)	N/A	0.031	0.027	0.080	0.286	0.248	2.523	0.274	15.911

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	27	28	0	34	28
N.S.	1	1.00	1.67	1.03	0.90	0.93	0.00	1.13	0.93
time (sec)	N/A	0.053	0.009	0.154	0.197	0.254	0.000	0.273	0.046

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	44	73	49	0	73	67
N.S.	1	1.00	0.74	0.72	1.20	0.80	0.00	1.20	1.10
time (sec)	N/A	0.048	0.075	0.219	0.290	0.272	0.000	0.279	16.433

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	49	43	45	0	57	43
N.S.	1	1.00	1.42	0.98	0.86	0.90	0.00	1.14	0.86
time (sec)	N/A	0.076	0.008	0.228	0.197	0.265	0.000	0.276	15.990

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	61	103	68	0	96	91
N.S.	1	1.00	0.76	0.69	1.16	0.76	0.00	1.08	1.02
time (sec)	N/A	0.060	0.079	0.256	0.279	0.252	0.000	0.280	16.137

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	113	46	304	33	0	0	42
N.S.	1	1.00	4.35	1.77	11.69	1.27	0.00	0.00	1.62
time (sec)	N/A	0.044	0.442	0.724	0.429	0.273	0.000	0.000	15.610

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	111	44	296	33	0	0	41
N.S.	1	1.00	4.44	1.76	11.84	1.32	0.00	0.00	1.64
time (sec)	N/A	0.044	0.299	0.340	0.428	0.257	0.000	0.000	15.212

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	84	291	0	142	0	0	0
N.S.	1	1.00	0.76	2.65	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.089	1.113	22.693	0.000	0.099	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	184	798	0	143	0	0	0
N.S.	1	1.00	1.67	7.25	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.093	1.201	5.330	0.000	0.098	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	220	0	111	0	0	0
N.S.	1	1.00	0.81	3.06	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.061	0.458	2.141	0.000	0.094	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	126	763	0	99	0	0	0
N.S.	1	1.00	1.85	11.22	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.060	0.795	3.781	0.000	0.088	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	218	0	100	0	0	0
N.S.	1	1.00	0.88	2.91	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.067	0.443	2.913	0.000	0.095	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	133	798	0	108	0	0	0
N.S.	1	1.00	1.73	10.36	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.069	1.218	5.274	0.000	0.091	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	79	291	0	119	0	0	0
N.S.	1	1.00	0.71	2.60	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.091	0.925	4.658	0.000	0.092	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	143	866	0	129	0	0	0
N.S.	1	1.00	1.28	7.73	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.091	1.325	7.760	0.000	0.101	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	41	0	19	0	46	21
N.S.	1	1.00	1.00	1.95	0.00	0.90	0.00	2.19	1.00
time (sec)	N/A	0.025	0.217	1.561	0.000	0.247	0.000	0.323	0.215

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	107	40	283	29	0	0	37
N.S.	1	1.00	5.10	1.90	13.48	1.38	0.00	0.00	1.76
time (sec)	N/A	0.033	0.386	1.148	0.428	0.249	0.000	0.000	15.392

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	41	42	19	0	22	42
N.S.	1	1.00	1.00	2.16	2.21	1.00	0.00	1.16	2.21
time (sec)	N/A	0.026	0.031	0.319	0.195	0.235	0.000	0.312	0.126

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	41	30	19	0	30	19
N.S.	1	1.00	1.00	2.16	1.58	1.00	0.00	1.58	1.00
time (sec)	N/A	0.024	0.035	0.240	0.203	0.251	0.000	0.296	15.172

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	36	33	19	0	23	23
N.S.	1	1.00	1.00	1.89	1.74	1.00	0.00	1.21	1.21
time (sec)	N/A	0.026	0.028	0.194	0.199	0.245	0.000	0.299	0.059

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	20	19	0	20	21
N.S.	1	1.00	1.00	1.68	1.05	1.00	0.00	1.05	1.11
time (sec)	N/A	0.027	0.032	0.138	0.202	0.241	0.000	0.285	15.139

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	22	19	0	24	22
N.S.	1	1.00	1.00	1.06	1.29	1.12	0.00	1.41	1.29
time (sec)	N/A	0.018	0.015	0.109	0.194	0.232	0.000	0.290	15.104

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	19	0	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.73	0.00	1.00	1.00
time (sec)	N/A	0.009	0.002	0.062	0.197	0.236	0.000	0.269	15.275

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	11	14	11	11
N.S.	1	1.00	2.09	1.09	1.00	1.00	1.27	1.00	1.00
time (sec)	N/A	0.005	0.014	0.078	0.199	0.251	0.059	0.266	0.022

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	23	17	49	14	14
N.S.	1	1.00	1.94	0.88	1.35	1.00	2.88	0.82	0.82
time (sec)	N/A	0.023	0.008	0.122	0.207	0.249	3.627	0.269	15.225

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	46	26	21	19	0	21	22
N.S.	1	1.00	2.42	1.37	1.11	1.00	0.00	1.11	1.16
time (sec)	N/A	0.026	0.035	0.150	0.232	0.250	0.000	0.276	0.046

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	31	29	33	19	0	23	19
N.S.	1	1.00	1.63	1.53	1.74	1.00	0.00	1.21	1.00
time (sec)	N/A	0.026	0.086	0.165	0.191	0.261	0.000	0.282	15.435

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	38	37	30	19	0	30	23
N.S.	1	1.00	2.00	1.95	1.58	1.00	0.00	1.58	1.21
time (sec)	N/A	0.028	0.030	0.204	0.207	0.251	0.000	0.287	15.531

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	59	73	95	99	0	164	153
N.S.	1	1.00	0.69	0.86	1.12	1.16	0.00	1.93	1.80
time (sec)	N/A	0.078	0.175	0.345	0.201	0.268	0.000	0.314	18.027

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	70	88	0	122	109
N.S.	1	1.00	0.95	0.95	1.11	1.40	0.00	1.94	1.73
time (sec)	N/A	0.061	0.121	0.256	0.208	0.275	0.000	0.298	17.120

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	58	74	0	107	85
N.S.	1	1.00	1.00	1.00	1.23	1.57	0.00	2.28	1.81
time (sec)	N/A	0.048	0.009	0.182	0.198	0.270	0.000	0.326	15.824

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	31	60	0	57	47
N.S.	1	1.00	1.00	1.25	1.29	2.50	0.00	2.38	1.96
time (sec)	N/A	0.021	0.007	0.100	0.194	0.264	0.000	0.278	15.308

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	37	36	0	43	57
N.S.	1	1.00	1.00	1.81	2.31	2.25	0.00	2.69	3.56
time (sec)	N/A	0.029	0.002	0.115	0.196	0.260	0.000	0.284	15.211

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	17	0	39	17
N.S.	1	1.00	1.73	1.07	1.33	1.13	0.00	2.60	1.13
time (sec)	N/A	0.034	0.004	0.121	0.195	0.260	0.000	0.285	15.416

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	32	34	29	0	82	31
N.S.	1	1.00	0.92	0.84	0.89	0.76	0.00	2.16	0.82
time (sec)	N/A	0.047	0.049	0.150	0.197	0.250	0.000	0.290	15.318

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	44	46	42	0	98	55
N.S.	1	1.00	1.06	0.81	0.85	0.78	0.00	1.81	1.02
time (sec)	N/A	0.053	0.046	0.184	0.214	0.256	0.000	0.292	15.615

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	57	57	53	0	140	75
N.S.	1	1.00	0.96	0.75	0.75	0.70	0.00	1.84	0.99
time (sec)	N/A	0.069	0.087	0.227	0.205	0.259	0.000	0.293	15.253

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	69	69	64	0	154	113
N.S.	1	1.00	0.97	0.75	0.75	0.70	0.00	1.67	1.23
time (sec)	N/A	0.071	0.076	0.275	0.216	0.255	0.000	0.287	19.245

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	102	555	0	207	0	0	0
N.S.	1	1.00	0.60	3.28	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.161	0.821	1.923	0.000	0.099	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	534	0	183	0	0	0
N.S.	1	1.00	0.67	3.96	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.127	0.527	2.878	0.000	0.094	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	73	460	0	143	0	0	0
N.S.	1	1.00	0.67	4.22	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.108	0.792	1.839	0.000	0.096	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	414	0	122	0	0	0
N.S.	1	1.00	0.67	4.87	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.092	0.350	2.864	0.000	0.094	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	81	536	0	150	0	0	0
N.S.	1	1.00	0.70	4.62	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.114	0.511	1.727	0.000	0.100	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	91	566	0	164	0	0	0
N.S.	1	1.00	0.62	3.85	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.143	1.237	2.811	0.000	0.105	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	582	0	175	0	0	0
N.S.	1	1.00	0.59	3.31	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.153	1.324	1.725	0.000	0.108	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	104	127	122	0	246	197
N.S.	1	1.00	0.66	0.85	1.04	1.00	0.00	2.02	1.61
time (sec)	N/A	0.116	0.485	0.474	0.215	0.271	0.000	0.309	17.674

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	106	139	117	0	230	160
N.S.	1	1.00	0.73	1.09	1.43	1.21	0.00	2.37	1.65
time (sec)	N/A	0.094	0.198	0.394	0.219	0.279	0.000	0.306	17.938

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	51	68	79	94	0	162	123
N.S.	1	1.00	0.65	0.87	1.01	1.21	0.00	2.08	1.58
time (sec)	N/A	0.096	0.165	0.261	0.205	0.265	0.000	0.299	17.097

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	63	75	82	0	115	89
N.S.	1	1.00	1.16	1.24	1.47	1.61	0.00	2.25	1.75
time (sec)	N/A	0.057	0.005	0.206	0.206	0.268	0.000	0.314	15.737

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	35	34	71	0	60	161
N.S.	1	1.00	1.00	1.30	1.26	2.63	0.00	2.22	5.96
time (sec)	N/A	0.026	0.001	0.026	0.205	0.264	0.000	0.294	14.946

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	37	46	45	0	70	68
N.S.	1	1.00	1.41	1.37	1.70	1.67	0.00	2.59	2.52
time (sec)	N/A	0.064	0.014	0.157	0.209	0.270	0.000	0.306	14.916

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	35	42	33	0	86	34
N.S.	1	1.00	1.31	0.83	1.00	0.79	0.00	2.05	0.81
time (sec)	N/A	0.077	0.037	0.151	0.195	0.246	0.000	0.288	14.848

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	49	55	45	0	138	66
N.S.	1	1.00	0.95	0.88	0.98	0.80	0.00	2.46	1.18
time (sec)	N/A	0.087	0.077	0.184	0.199	0.272	0.000	0.297	14.711

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	63	77	65	0	200	81
N.S.	1	1.00	0.80	0.72	0.88	0.74	0.00	2.27	0.92
time (sec)	N/A	0.098	0.124	0.239	0.206	0.254	0.000	0.286	15.058

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	79	89	73	0	222	104
N.S.	1	1.00	0.89	0.81	0.91	0.74	0.00	2.27	1.06
time (sec)	N/A	0.139	0.128	0.282	0.196	0.272	0.000	0.308	14.919

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	102	95	115	93	0	284	126
N.S.	1	1.00	0.77	0.72	0.87	0.70	0.00	2.15	0.95
time (sec)	N/A	0.117	0.217	0.342	0.219	0.279	0.000	0.297	15.013

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	308	949	0	225	0	0	0
N.S.	1	1.00	1.73	5.33	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.182	5.708	2.950	0.000	0.105	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	302	611	0	193	0	0	0
N.S.	1	1.00	2.22	4.49	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.159	2.201	1.528	0.000	0.093	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	156	847	0	153	0	0	0
N.S.	1	1.00	1.42	7.70	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.132	1.394	2.734	0.000	0.094	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	143	608	0	160	0	0	0
N.S.	1	1.00	1.22	5.20	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.153	1.721	1.594	0.000	0.098	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	169	956	0	176	0	0	0
N.S.	1	1.00	1.13	6.37	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.172	2.198	2.692	0.000	0.102	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	177	726	0	193	0	0	0
N.S.	1	1.00	0.96	3.92	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.201	2.600	1.543	0.000	0.106	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [39] had the largest ratio of [.2308000000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	4	3	1.00	21	0.143
3	A	3	2	1.00	21	0.095
4	A	3	3	1.00	21	0.143
5	A	3	3	1.00	21	0.143
6	A	2	2	1.00	19	0.105
7	A	3	2	1.00	12	0.167
8	A	2	2	1.00	19	0.105
9	A	2	2	1.00	21	0.095
10	A	3	2	1.00	21	0.095
11	A	3	3	1.00	21	0.143
12	A	4	3	1.00	21	0.143
13	A	4	3	1.00	21	0.143
14	A	1	1	1.00	29	0.034
15	A	1	1	1.00	28	0.036
16	A	4	4	1.00	25	0.160
17	A	4	4	1.00	25	0.160
18	A	3	3	1.00	25	0.120
19	A	3	3	1.00	25	0.120
20	A	3	3	1.00	25	0.120
21	A	3	3	1.00	25	0.120
22	A	4	4	1.00	25	0.160
23	A	4	4	1.00	25	0.160
24	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	1	1	1.00	24	0.042
26	A	1	1	1.00	21	0.048
27	A	1	1	1.00	21	0.048
28	A	1	1	1.00	21	0.048
29	A	1	1	1.00	21	0.048
30	A	1	1	1.00	19	0.053
31	A	2	2	1.00	10	0.200
32	A	1	1	1.00	8	0.125
33	A	1	1	1.00	19	0.053
34	A	1	1	1.00	21	0.048
35	A	1	1	1.00	21	0.048
36	A	1	1	1.00	21	0.048
37	A	7	5	1.00	28	0.179
38	A	6	5	1.00	28	0.179
39	A	6	6	1.00	26	0.231
40	A	4	3	1.00	19	0.158
41	A	4	4	1.00	26	0.154
42	A	3	3	1.00	28	0.107
43	A	5	5	1.00	28	0.179
44	A	6	5	1.00	28	0.179
45	A	7	5	1.00	28	0.179
46	A	7	5	1.00	28	0.179
47	A	10	7	1.00	32	0.219
48	A	9	7	1.00	32	0.219
49	A	8	7	1.00	32	0.219
50	A	7	6	1.00	32	0.188
51	A	8	7	1.00	32	0.219
52	A	9	7	1.00	32	0.219
53	A	10	7	1.00	32	0.219
54	A	7	5	1.00	29	0.172
55	A	6	5	1.00	29	0.172
56	A	6	6	1.00	29	0.207
57	A	5	5	1.00	27	0.185
58	A	4	3	1.00	20	0.150
59	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	4	1.00	29	0.138
61	A	6	5	1.00	29	0.172
62	A	6	5	1.00	29	0.172
63	A	8	6	1.00	29	0.207
64	A	7	5	1.00	29	0.172
65	A	8	6	1.00	33	0.182
66	A	7	6	1.00	33	0.182
67	A	6	5	1.00	33	0.152
68	A	6	5	1.00	33	0.152
69	A	7	6	1.00	33	0.182
70	A	8	6	1.00	33	0.182

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(7A+6C)\tan(c+dx)}{7d} + \frac{C\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{2(7A+6C)\tan^3(c+dx)}{21d} + \frac{(7A+6C)\tan^5(c+dx)}{35d}$$

```
[Out] 1/7*(7*A+6*C)*tan(d*x+c)/d+1/7*C*sec(d*x+c)^6*tan(d*x+c)/d+2/21*(7*A+6*C)*t
an(d*x+c)^3/d+1/35*(7*A+6*C)*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4131, 3852}

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(7A+6C)\tan^5(c+dx)}{35d} + \frac{2(7A+6C)\tan^3(c+dx)}{21d} + \frac{(7A+6C)\tan(c+dx)}{7d} + \frac{C\tan(c+dx)\sec^6(c+dx)}{7d}$$

```
[In] Int[Sec[c + d*x]^6*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((7*A + 6*C)*Tan[c + d*x])/(7*d) + (C*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) +
(2*(7*A + 6*C)*Tan[c + d*x]^3)/(21*d) + ((7*A + 6*C)*Tan[c + d*x]^5)/(35*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{C\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{1}{7}(7A+6C)\int\sec^6(c+dx)\,dx\\
&= \frac{C\sec^6(c+dx)\tan(c+dx)}{7d}-\frac{(7A+6C)\text{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{7d}\\
&= \frac{(7A+6C)\tan(c+dx)}{7d}+\frac{C\sec^6(c+dx)\tan(c+dx)}{7d}\\
&\quad+\frac{2(7A+6C)\tan^3(c+dx)}{21d}+\frac{(7A+6C)\tan^5(c+dx)}{35d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int\sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\
&=\frac{A\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}\\
&\quad+\frac{C\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^6*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (A*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d + (C*(Tan[c
+ d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-C\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-C\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}$
parts	$-\frac{A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}$
risch	$\frac{16i\left(70A\,e^{8i(dx+c)}+175A\,e^{6i(dx+c)}+210C\,e^{6i(dx+c)}+147A\,e^{4i(dx+c)}+126C\,e^{4i(dx+c)}+49A\,e^{2i(dx+c)}+42C\,e^{2i(dx+c)}+7A\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$
parallelrisch	$\frac{(1176A+1008C)\sin(3dx+3c)+(392A+336C)\sin(5dx+5c)+(56A+48C)\sin(7dx+7c)+840\sin(dx+c)(A+2C)}{105d(\cos(7dx+7c)+7\cos(5dx+5c)+21\cos(3dx+3c)+35\cos(dx+c))}$
norman	$\frac{-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}+\frac{4(5A+3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4(5A+3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3d}+\frac{8(91A+53C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^7}$

```
[In] int(sec(d*x+c)^6*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-C*(-16/35-1/7
*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\left(8\,(7\,A+6\,C)\cos{(dx+c)}^6+4\,(7\,A+6\,C)\cos{(dx+c)}^4+3\,(7\,A+6\,C)\cos{(dx+c)}^2+15\,C\right)\sin{(dx+c)}}{105\,d\cos{(dx+c)}^7}$$

```
[In] integrate(sec(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*A + 6*C)*cos(d*x + c)^6 + 4*(7*A + 6*C)*cos(d*x + c)^4 + 3*(7*A
 + 6*C)*cos(d*x + c)^2 + 15*C)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(A+C\sec^2\left(c+dx\right)\right)\sec^6\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**6*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{15\,C\tan\left(dx+c\right)^7+21\left(A+3\,C\right)\tan\left(dx+c\right)^5+35\left(2\,A+3\,C\right)\tan\left(dx+c\right)^3+105\left(A+C\right)\tan\left(dx+c\right.}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*C*tan(d*x + c)^7 + 21*(A + 3*C)*tan(d*x + c)^5 + 35*(2*A + 3*C)*t
an(d*x + c)^3 + 105*(A + C)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \sec^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{15\,C\tan\left(dx+c\right)^7+21\,A\tan\left(dx+c\right)^5+63\,C\tan\left(dx+c\right)^5+70\,A\tan\left(dx+c\right)^3+105\,C\tan\left(dx+c\right)^3+}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*C*tan(d*x + c)^7 + 21*A*tan(d*x + c)^5 + 63*C*tan(d*x + c)^5 + 70
*A*tan(d*x + c)^3 + 105*C*tan(d*x + c)^3 + 105*A*tan(d*x + c) + 105*C*tan(d
*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sec^6(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{\frac{C\tan(c+d\,x)^7}{7}+\left(\frac{A}{5}+\frac{3\,C}{5}\right)\,\tan(c+d\,x)^5+\left(\frac{2\,A}{3}+C\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x)^6,x)
[Out] (tan(c + d*x)^3*((2*A)/3 + C) + (C*tan(c + d*x)^7)/7 + tan(c + d*x)*(A + C)
 + tan(c + d*x)^5*(A/5 + (3*C)/5))/d
```

3.2 $\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(6A+5C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(6A+5C)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(6A+5C)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{C\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[Out] 1/16*(6*A+5*C)*arctanh(sin(d*x+c))/d+1/16*(6*A+5*C)*sec(d*x+c)*tan(d*x+c)/d
+1/24*(6*A+5*C)*sec(d*x+c)^3*tan(d*x+c)/d+1/6*C*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3853, 3855}

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(6A+5C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(6A+5C)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(6A+5C)\tan(c+dx)\sec(c+dx)}{16d}+\frac{C\tan(c+dx)\sec^5(c+dx)}{6d}$$

```
[In] Int[Sec[c + d*x]^5*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((6*A + 5*C)*ArcTanh[Sin[c + d*x]])/(16*d) + ((6*A + 5*C)*Sec[c + d*x]*Tan[
c + d*x])/(16*d) + ((6*A + 5*C)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (C*Se
c[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}(6A+5C)\int \sec^5(c+dx)\,dx\\
&= \frac{(6A+5C)\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad+\frac{C\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{8}(6A+5C)\int \sec^3(c+dx)\,dx\\
&= \frac{(6A+5C)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(6A+5C)\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad+\frac{C\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{16}(6A+5C)\int \sec(c+dx)\,dx\\
&= \frac{(6A+5C)\text{arctanh}(\sin(c+dx))}{16d}+\frac{(6A+5C)\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad+\frac{(6A+5C)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{C\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{3A\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{5C\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{3A\sec(c+dx)\tan(c+dx)}{8d}+\frac{5C\sec(c+dx)\tan(c+dx)}{16d}+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{5C\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{C\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[In] Integrate[Sec[c + d*x]^5*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*A*ArcTanh[Sin[c + d*x]])/(8*d) + (5*C*ArcTanh[Sin[c + d*x]])/(16*d) + (3
*A*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (5*C*Sec[c + d*x]*Tan[c + d*x])/(16*d
) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (5*C*Sec[c + d*x]^3*Tan[c + d*x
])/(24*d) + (C*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx\cdots}{16}\right.\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx\cdots}{16}\right.\right.}{d}$
parts	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{C\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(\cdots}{}\right.\right.}{}$
parallelrisch	$\frac{-270\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\left(A+\frac{5C}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+270\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cdots\right.}{48d(\cos(6dx+6c)+6\cos\cdots}$
norman	$\frac{\frac{(2A+15C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}+\frac{(2A+15C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(10A+11C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{(10A+11C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}-\frac{(42A-5C)\tan\left(\frac{dx}{2}+\cdots\right.}{24d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^6}$
risch	$-\frac{i\,e^{i(dx+c)}\left(18A\,e^{10i(dx+c)}+15C\,e^{10i(dx+c)}+102A\,e^{8i(dx+c)}+85C\,e^{8i(dx+c)}+84A\,e^{6i(dx+c)}+198C\,e^{6i(dx+c)}-84A\,e^{4i(\cdots}\right.}{24d\left(e^{2i(dx+c)}+1\right)^6}$

```
[In] int(sec(d*x+c)^5*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+C*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*
x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{3\,(6\,A+5\,C)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)}-3\,(6\,A+5\,C)\cos{(dx+c)}^6\log{(-\sin{(dx+c)}+1)}+2\,(}{96\,d\cos{(dx+c)}^6}$$

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/96*(3*(6*A + 5*C)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(6*A + 5*C)*co
s(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(6*A + 5*C)*cos(d*x + c)^4 + 2*(
6*A + 5*C)*cos(d*x + c)^2 + 8*C)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(A+C\sec^2{(c+dx)}\right)\sec^5{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**5*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{3\,(6\,A+5\,C)\log{(\sin{(dx+c)}+1)}-3\,(6\,A+5\,C)\log{(\sin{(dx+c)}-1)}-\frac{2\left(3\,(6\,A+5\,C)\sin(dx+c)^5-8\,(6\,A+5\,C)\sin\right.}{\sin(dx+c)^6-3\,\sin(dx+c)}}{96\,d}$$

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/96*(3*(6*A + 5*C)*log(sin(d*x + c) + 1) - 3*(6*A + 5*C)*log(sin(d*x + c)
- 1) - 2*(3*(6*A + 5*C)*sin(d*x + c)^5 - 8*(6*A + 5*C)*sin(d*x + c)^3 + 3*(
10*A + 11*C)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x +
 c)^2 - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{3\,(6\,A+5\,C)\log\left(|\sin\left(dx+c\right)+1|\right)-3\,(6\,A+5\,C)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(18\,A\sin(dx+c)^5+15\,C\sin(dx+c)^5}{}}{96\,d}$$

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*A + 5*C)*log(abs(sin(d*x + c) + 1)) - 3*(6*A + 5*C)*log(abs(sin(
d*x + c) - 1)) - 2*(18*A*sin(d*x + c)^5 + 15*C*sin(d*x + c)^5 - 48*A*sin(d*
x + c)^3 - 40*C*sin(d*x + c)^3 + 30*A*sin(d*x + c) + 33*C*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\operatorname{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right)}{d}-\frac{\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right)\,\sin\left(c+d\,x\right)^5+\left(-A-\frac{5\,C}{6}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{5\,A}{8}+\frac{11\,C}{16}\right)\,\sin\left(c+d\,x\right)}{d\,\left(\sin\left(c+d\,x\right)^6-3\sin\left(c+d\,x\right)^4+3\sin\left(c+d\,x\right)^2-1\right)}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x)^5,x)
```

```
[Out] (atanh(sin(c + d*x))*((3*A)/8 + (5*C)/16))/d - (sin(c + d*x)*((5*A)/8 + (11
*C)/16) - sin(c + d*x)^3*(A + (5*C)/6) + sin(c + d*x)^5*((3*A)/8 + (5*C)/16
))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))
```

3.3 $\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(5A+4C)\tan(c+dx)}{5d} + \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(5A+4C)\tan^3(c+dx)}{15d}$$

```
[Out] 1/5*(5*A+4*C)*tan(d*x+c)/d+1/5*C*sec(d*x+c)^4*tan(d*x+c)/d+1/15*(5*A+4*C)*t
an(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4131, 3852}

$$\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(5A+4C)\tan^3(c+dx)}{15d} + \frac{(5A+4C)\tan(c+dx)}{5d} + \frac{C\tan(c+dx)\sec^4(c+dx)}{5d}$$

```
[In] Int[Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((5*A + 4*C)*Tan[c + d*x])/(5*d) + (C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) +
((5*A + 4*C)*Tan[c + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}(5A+4C)\int \sec^4(c+dx)\,dx \\
&= \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} - \frac{(5A+4C)\text{Subst}\left(\int (1+x^2)\;dx, x, -\tan(c+dx)\right)}{5d} \\
&= \frac{(5A+4C)\tan(c+dx)}{5d} + \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(5A+4C)\tan^3(c+dx)}{15d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\;dx = \frac{A\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d} + \frac{C\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2),x]
[Out] (A*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d + (C*(Tan[c + d*x] + (2*Tan[c + d*x
]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$-\frac{A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{(50A+40C)\sin(3dx+3c)+(10A+8C)\sin(5dx+5c)+40\sin(dx+c)(A+2C)}{15d(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))}$
risch	$\frac{4i\left(15A\,e^{6i(dx+c)}+35A\,e^{4i(dx+c)}+40C\,e^{4i(dx+c)}+25A\,e^{2i(dx+c)}+20C\,e^{2i(dx+c)}+5A+4C\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{8(2A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{8(2A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{4(25A+29C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^5}$

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-C*(-8/15-1/5*sec(d*x+c)^4-4/15*s
ec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{\left(2\,(5\,A+4\,C)\cos\left(dx+c\right)^4+(5\,A+4\,C)\cos\left(dx+c\right)^2+3\,C\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/15*(2*(5*A + 4*C)*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 3*C)*sin(
d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \int \left(A+C\sec^2\left(c+dx\right)\right)\sec^4\left(c+dx\right) dx$$

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \sec^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{3\,C\tan\left(dx+c\right)^5+5\left(A+2\,C\right)\tan\left(dx+c\right)^3+15\left(A+C\right)\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*C*tan(d*x + c)^5 + 5*(A + 2*C)*tan(d*x + c)^3 + 15*(A + C)*tan(d*x
+ c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sec^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{3\,C\tan\left(dx+c\right)^5+5\,A\tan\left(dx+c\right)^3+10\,C\tan\left(dx+c\right)^3+15\,A\tan\left(dx+c\right)+15\,C\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*C*tan(d*x + c)^5 + 5*A*tan(d*x + c)^3 + 10*C*tan(d*x + c)^3 + 15*A*
tan(d*x + c) + 15*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\frac{C\tan(c+d\,x)^5}{5}+\left(\frac{A}{3}+\frac{2\,C}{3}\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x)^4,x)
```

```
[Out] ((C*tan(c + d*x)^5)/5 + tan(c + d*x)*(A + C) + tan(c + d*x)^3*(A/3 + (2*C)/
3))/d
```

3.4 $\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(4A+3C)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 1/8*(4*A+3*C)*arctanh(sin(d*x+c))/d+1/8*(4*A+3*C)*sec(d*x+c)*tan(d*x+c)/d+1
/4*C*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3853, 3855}

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(4A+3C)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{C\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[In] Int[Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*A + 3*C)*Sec[c + d*x]*Tan[c
 + d*x])/(8*d) + (C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{C\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(4A+3C)\int \sec^3(c+dx)\,dx \\ &= \frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d} \\ &\quad + \frac{C\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{8}(4A+3C)\int \sec(c+dx)\,dx \\ &= \frac{(4A+3C)\text{arctanh}(\sin(c+dx))}{8d} \\ &\quad + \frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{C\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{2d} + \frac{3C\text{arctanh}(\sin(c+dx))}{8d} + \frac{A\sec(c+dx)\tan(c+dx)}{2d} + \frac{3C\sec(c+dx)\tan(c+dx)}{8d} + \frac{C\sec^3(c+dx)\tan(c+dx)}{4d}$$

[In] Integrate[Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2),x]

[Out] (A*ArcTanh[Sin[c + d*x]])/(2*d) + (3*C*ArcTanh[Sin[c + d*x]])/(8*d) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (3*C*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx}{8}\right.}{d}$
default	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx}{8}\right.}{d}$
parts	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan}{8}\right.}{d}$
parallelrisch	$\frac{-8\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{3C}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{3C}{4}\right)\ln\left(\tan\right.}{4d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
norman	$\frac{-\frac{(4A-3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}-\frac{(4A-3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}+\frac{(4A+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(4A+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}-\frac{(4A+3C)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{8d}$
risch	$-\frac{ie^{i(dx+c)}\left(4A\,e^{6i(dx+c)}+3C\,e^{6i(dx+c)}+4A\,e^{4i(dx+c)}+11C\,e^{4i(dx+c)}-4A\,e^{2i(dx+c)}-11C\,e^{2i(dx+c)}-4A-3C\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln(e}{}$

[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(4A+3C)\cos(dx+c)^4\log(\sin(dx+c)+1)-(4A+3C)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left((4\right.}{16\,d\cos(dx+c)^4}$$

[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")

[Out] 1/16*((4*A + 3*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*A + 3*C)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((4*A + 3*C)*cos(d*x + c)^2 + 2*C)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F]

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(A+C\sec^2{(c+dx)}\right)\sec^3{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(4A+3C)\log\left(\sin\left(dx+c\right)+1\right)-(4A+3C)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((4A+3C)\sin(dx+c)^3-(4A+5C)\sin(dx+c}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/16*((4*A + 3*C)*log(sin(d*x + c) + 1) - (4*A + 3*C)*log(sin(d*x + c) - 1)
 - 2*((4*A + 3*C)*sin(d*x + c)^3 - (4*A + 5*C)*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(4A+3C)\log\left(|\sin\left(dx+c\right)+1|\right)-(4A+3C)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(4A\sin(dx+c)^3+3C\sin(dx+c)^3-4A\sin(}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/16*((4*A + 3*C)*log(abs(sin(d*x + c) + 1)) - (4*A + 3*C)*log(abs(sin(d*x
+ c) - 1)) - 2*(4*A*sin(d*x + c)^3 + 3*C*sin(d*x + c)^3 - 4*A*sin(d*x + c)
- 5*C*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{\sin\left(c+d\,x\right)\,\left(\frac{A}{2}+\frac{5\,C}{8}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{A}{2}+\frac{3\,C}{8}\right)}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}+\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{A}{2}+\frac{3\,C}{8}\right)}{d}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x)^3,x)
[Out] (sin(c + d*x)*(A/2 + (5*C)/8) - sin(c + d*x)^3*(A/2 + (3*C)/8))/(d*(sin(c + 
 d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(sin(c + d*x))*(A/2 + (3*C)/8))/d
```

3.5 $\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(3A+2C)\tan(c+dx)}{3d}+\frac{C\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] 1/3*(3*A+2*C)*tan(d*x+c)/d+1/3*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4131, 3852, 8}

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(3A+2C)\tan(c+dx)}{3d}+\frac{C\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[In] Int[Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((3*A + 2*C)*Tan[c + d*x])/(3*d) + (C*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}(3A+2C)\int \sec^2(c+dx)\,dx \\
&= \frac{C\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{(3A+2C)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d} \\
&= \frac{(3A+2C)\tan(c+dx)}{3d} + \frac{C\sec^2(c+dx)\tan(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\tan(c+dx)}{d} + \frac{C\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2),x]
[Out] (A*Tan[c + d*x])/d + (C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{A\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$	35
default	$\frac{A\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$	35
parts	$\frac{A\tan(dx+c)}{d}-\frac{C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$	37
parallelrisch	$\frac{(3A+2C)\sin(3dx+3c)+3\sin(dx+c)(A+2C)}{3d(\cos(3dx+3c)+3\cos(dx+c))}$	57
risch	$\frac{2i\left(3A\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+3A+2C\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	63
norman	$\frac{-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4(3A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$	75

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*tan(d*x+c)-C*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{\left((3\,A+2\,C)\cos\left(dx+c\right)^2+C\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*((3*A + 2*C)*cos(d*x + c)^2 + C)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(A+C\sec^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)C+3\,A\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*C + 3*A*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{C\tan\left(dx+c\right)^3+3\,A\tan\left(dx+c\right)+3\,C\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/3*(C*tan(d*x + c)^3 + 3*A*tan(d*x + c) + 3*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{C\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(A+C\right)}{d}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x)^2,x)
[Out] (C*tan(c + d*x)^3)/(3*d) + (tan(c + d*x)*(A + C))/d
```

3.6 $\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{C\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(2*A+C)*arctanh(sin(d*x+c))/d+1/2*C*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4131, 3855}

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{C\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}(2A+C)\int \sec(c+dx)\,dx \\ &= \frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\text{arctanh}(\sin(c+dx))}{d} + \frac{C\text{arctanh}(\sin(c+dx))}{2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]*(A + C*Sec[c + d*x]^2),x]
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	55
default	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	55
parts	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	57
parallelrisch	$\frac{-\left(A+\frac{C}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(A+\frac{C}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+C\sin(dx+c)}{d(1+\cos(2dx+2c))}$	86
norman	$\frac{\frac{C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}-\frac{(2A+C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2A+C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	93
risch	$-\frac{iC\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)A}{d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)C}{2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)A}{d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)C}{2d}$	118

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*ln(sec(d*x+c)+tan(d*x+c))+C*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*
x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(2A+C)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2A+C)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2C\sin(d}{4d\cos(dx+c)^2}$$

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/4*((2*A + C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A + C)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(A+C\sec^2{(c+dx)}\right)\sec{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(2A+C)\log{(\sin{(dx+c)}+1)}-(2A+C)\log{(\sin{(dx+c)}-1)}-\frac{2C\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/4*((2*A + C)*log(sin(d*x + c) + 1) - (2*A + C)*log(sin(d*x + c) - 1) - 2*
C*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(2A+C)\log{(|\sin{(dx+c)}+1|)}-(2A+C)\log{(|\sin{(dx+c)}-1|)}-\frac{2C\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/4*((2*A + C)*log(abs(sin(d*x + c) + 1)) - (2*A + C)*log(abs(sin(d*x + c)
- 1)) - 2*C*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 16.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{\operatorname{atanh}(\sin(c+d\,x))\,\left(A+\frac{C}{2}\right)}{d}-\frac{C\,\sin(c+d\,x)}{2\,d\,\left(\sin(c+d\,x)^2-1\right)}$$

```
[In] int((A + C/cos(c + d*x)^2)/cos(c + d*x),x)
[Out] (atanh(sin(c + d*x))*(A + C/2))/d - (C*sin(c + d*x))/(2*d*(sin(c + d*x)^2 -
 1))
```

3.7 $\int (A + C\sec^2(c + dx))\, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (A + C\sec^2(c + dx))\, dx = Ax + \frac{C\tan(c + dx)}{d}$$

```
[Out] A*x+C*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$\int (A + C\sec^2(c + dx))\, dx = Ax + \frac{C\tan(c + dx)}{d}$$

```
[In] Int[A + C*Sec[c + d*x]^2,x]
```

```
[Out] A*x + (C*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= Ax + C\int \sec^2(c+dx)\,dx\\ &= Ax - \frac{C\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= Ax + \frac{C\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(A + C\sec^2(c+dx)\right)\,dx = Ax + \frac{C\tan(c+dx)}{d}$$

```
[In] Integrate[A + C*Sec[c + d*x]^2,x]
[Out] A*x + (C*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$Ax + \frac{C\tan(dx+c)}{d}$	16
parts	$Ax + \frac{C\tan(dx+c)}{d}$	16
derivativedivides	$\frac{(dx+c)A + C\tan(dx+c)}{d}$	21
risch	$Ax + \frac{2iC}{d\left(e^{2i(dx+c)}+1\right)}$	25
parallelrisch	$-\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)} + Ax$	35
norman	$\frac{Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - Ax - \frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}$	51

```
[In] int(A+C*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] A*x+C*tan(d*x+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(A + C\sec^2(c + dx)\right)\,dx = \frac{Adx\cos\left(dx + c\right) + C\sin\left(dx + c\right)}{d\cos\left(dx + c\right)}$$

```
[In] integrate(A+C*sec(d*x+c)^2,x, algorithm="fricas")
[Out] (A*d*x*cos(d*x + c) + C*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \left(A + C\sec^2(c + dx)\right)\,dx = \int \left(A + C\sec^2\left(c + dx\right)\right)\,dx$$

```
[In] integrate(A+C*sec(d*x+c)**2,x)
[Out] Integral(A + C*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(A + C\sec^2(c + dx)\right)\,dx = Ax + \frac{C\tan\left(dx + c\right)}{d}$$

```
[In] integrate(A+C*sec(d*x+c)^2,x, algorithm="maxima")
[Out] A*x + C*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(A + C\sec^2(c + dx)\right)\,dx = Ax + \frac{C\tan\left(dx + c\right)}{d}$$

```
[In] integrate(A+C*sec(d*x+c)^2,x, algorithm="giac")
[Out] A*x + C*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(A + C\sec^2(c + dx)\right) \, dx = \frac{C\tan(c + d\,x) + A\,d\,x}{d}$$

```
[In] int(A + C/cos(c + d*x)^2,x)
[Out] (C*tan(c + d*x) + A*d*x)/d
```

3.8 $\int \cos(c+dx) \left(A+C\sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \cos(c+dx) \left(A+C\sec^2(c+dx)\right) dx = \frac{C\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{A\sin(c+dx)}{d}$$

```
[Out] C*arctanh(sin(d*x+c))/d+A*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4130, 3855}

$$\int \cos(c+dx) \left(A+C\sec^2(c+dx)\right) dx = \frac{A\sin(c+dx)}{d}+\frac{C\operatorname{arctanh}(\sin(c+dx))}{d}$$

```
[In] Int[Cos[c + d*x]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/d + (A*Sin[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{A\sin(c+dx)}{d} + C\int \sec(c+dx)\,dx\\ &= \frac{C\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{A\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \cos(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{C\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{A\cos(dx)\sin(c)}{d} + \frac{A\cos(c)\sin(dx)}{d}$$

```
[In] Integrate[Cos[c + d*x]*(A + C*Sec[c + d*x]^2),x]
[Out] (C*ArcTanh[Sin[c + d*x]])/d + (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{A\sin(dx+c)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
default	$\frac{A\sin(dx+c)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
parallelrisch	$\frac{A\sin(dx+c)-C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	43
risch	$-\frac{iA\,e^{i(dx+c)}}{2d}+\frac{iA\,e^{-i(dx+c)}}{2d}+\frac{\ln(e^{i(dx+c)}+i)C}{d}-\frac{\ln(e^{i(dx+c)}-i)C}{d}$	71
norman	$\frac{-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}+\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	101

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*sin(d*x+c)+C*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(c+dx) \left(A + C\sec^2(c+dx)\right) \, dx = \frac{C\log\left(\sin\left(dx+c\right)+1\right) - C\log\left(-\sin\left(dx+c\right)+1\right) + 2\,A\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(C*log(sin(d*x + c) + 1) - C*log(-sin(d*x + c) + 1) + 2*A*sin(d*x + c))
/d
```

Sympy [F]

$$\int \cos(c+dx) \left(A + C\sec^2(c+dx)\right) \, dx = \int \left(A + C\sec^2\left(c+dx\right)\right)\cos\left(c+dx\right) dx$$

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(c+dx) \left(A + C\sec^2(c+dx)\right) \, dx = \frac{C(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,A\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(C*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*sin(d*x + c))/
d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{C\log\left(|\sin\left(dx+c\right)+1|\right)-C\log\left(|\sin\left(dx+c\right)-1|\right)+2\,A\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(C*log(abs(sin(d*x + c) + 1)) - C*log(abs(sin(d*x + c) - 1)) + 2*A*sin(
d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\,\sin\left(c+d\,x\right)+C\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{d}$$

```
[In] int(cos(c + d*x)*(A + C/cos(c + d*x)^2),x)
```

```
[Out] (A*sin(c + d*x) + C*atanh(sin(c + d*x)))/d
```

3.9 $\int \cos^2(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \cos^2(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{1}{2}(A+2C)x+\frac{A\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(A+2*C)*x+1/2*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4130, 8}

$$\int \cos^2(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{A\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(A+2C)$$

```
[In] Int[Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((A + 2*C)*x)/2 + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{A\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(A+2C)\int 1\,dx \\ &= \frac{1}{2}(A+2C)x + \frac{A\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = Cx + \frac{A(c+dx)}{2d} + \frac{A\sin(2(c+dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2),x]
[Out] C*x + (A*(c + d*x))/(2*d) + (A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
risch	$\frac{Ax}{2}+Cx+\frac{A\sin(2dx+2c)}{4d}$
parallelrisch	$\frac{A\sin(2dx+2c)+2(A+2C)xd}{4d}$
derivativedivides	$\frac{A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C(dx+c)}{d}$
default	$\frac{A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C(dx+c)}{d}$
norman	$\frac{\left(-\frac{A}{2}-C\right)x+\left(-\frac{A}{2}-C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{A}{2}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{A}{2}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2A\tan\left(\frac{dx}{2}+\right.}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*A*x+C*x+1/4*A/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(A+2\,C)dx+A\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*((A + 2*C)*d*x + A*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=A\left(\begin{cases}\frac{x\sin^2{(c+dx)}}{2}+\frac{x\cos^2{(c+dx)}}{2}+\frac{\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d\neq 0\\ x\cos^2{(c)} & \text{otherwise}\end{cases}\right)+Cx$$

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2),x)
[Out] A*Piecewise((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c
 + d*x)/(2*d), Ne(d, 0)), (x*cos(c)**2, True)) + C*x
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(dx+c)(A+2\,C)+\frac{A\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*(A + 2*C) + A*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(dx+c)(A+2\,C)+\frac{A\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*((d*x + c)*(A + 2*C) + A*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{\frac{A\,\sin(2\,c+2\,d\,x)}{4}+d\,x\,\left(\frac{A}{2}+C\right)}{d}$$

```
[In] int(cos(c + d*x)^2*(A + C/cos(c + d*x)^2),x)
[Out] ((A*sin(2*c + 2*d*x))/4 + d*x*(A/2 + C))/d
```

3.10 $\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*A*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4129, 3092}

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
```

```
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cos(c+dx)\left(C+A\cos^2(c+dx)\right)\,dx\\ &= -\frac{\text{Subst}\left(\int \left(A+C-Ax^2\right)\,dx, x, -\sin(c+dx)\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{C\cos(dx)\sin(c)}{d}+\frac{C\cos(c)\sin(dx)}{d}+\frac{A\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2),x]
[Out] (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/d + (A*Sin[c + d*x])/d - (A*Sin
[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{A\sin(3dx+3c)+9\left(A+\frac{4C}{3}\right)\sin(dx+c)}{12d}$	31
derivativedivides	$\frac{\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\sin(dx+c)}{d}$	33
default	$\frac{\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\sin(dx+c)}{d}$	33
risch	$\frac{3A\sin(dx+c)}{4d}+\frac{C\sin(dx+c)}{d}+\frac{A\sin(3dx+3c)}{12d}$	40
norman	$\frac{\frac{2(A-3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2(A-3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	111

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*(A*sin(3*d*x+3*c)+9*(A+4/3*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{\left(A\cos\left(dx+c\right)^2+2\,A+3\,C\right)\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*(A*cos(d*x + c)^2 + 2*A + 3*C)*sin(d*x + c)/d
```

Sympy [F]

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(A+C\sec^2\left(c+dx\right)\right)\cos^3\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = -\frac{A\sin\left(dx+c\right)^3-3\left(A+C\right)\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*(A*sin(d*x + c)^3 - 3*(A + C)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = -\frac{A\sin\left(dx+c\right)^3-3\,A\sin\left(dx+c\right)-3\,C\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] -1/3*(A*sin(d*x + c)^3 - 3*A*sin(d*x + c) - 3*C*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=-\frac{\frac{A\sin(c+d\,x)^3}{3}-\sin\left(c+d\,x\right)\,\left(A+C\right)}{d}$$

```
[In] int(cos(c + d*x)^3*(A + C/cos(c + d*x)^2),x)
[Out] -((A*sin(c + d*x)^3)/3 - sin(c + d*x)*(A + C))/d
```

3.11 $\int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{1}{8}(3A+4C)x+\frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{A\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(3*A+4*C)*x+1/8*(3*A+4*C)*cos(d*x+c)*sin(d*x+c)/d+1/4*A*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4130, 2715, 8}

$$\int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{A\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{8}x(3A+4C)$$

```
[In] Int[Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((3*A + 4*C)*x)/8 + ((3*A + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*Cos[
c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3A+4C)\int\cos^2(c+dx)\,dx \\
&= \frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3A+4C)\int 1\,dx \\
&= \frac{1}{8}(3A+4C)x + \frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{4(3A+4C)(c+dx)+8(A+C)\sin(2(c+dx))+A\sin(4(c+dx))}{32d}$$

```
[In] Integrate[Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2),x]
[Out] (4*(3*A + 4*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + A*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8A+8C)\sin(2dx+2c)+A\sin(4dx+4c)+12d\left(A+\frac{4C}{3}\right)x}{32d}$
risch	$\frac{3Ax}{8}+\frac{Cx}{2}+\frac{A\sin(4dx+4c)}{32d}+\frac{A\sin(2dx+2c)}{4d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(-\frac{3A}{8}-\frac{C}{2}\right)x+\left(-\frac{9A}{8}-\frac{3C}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{3A}{4}-C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{3A}{4}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3A}{8}+\frac{C}{2}\right)x\tan\left(\right.}{\left(1+\tan\left(\right.\right.}$

[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/32*((8*A+8*C)*sin(2*d*x+2*c)+A*sin(4*d*x+4*c)+12*d*(A+4/3*C)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(3\,A+4\,C)dx+\left(2\,A\cos\left(dx+c\right)^3+(3\,A+4\,C)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="fricas")

[Out] 1/8*((3*A + 4*C)*d*x + (2*A*cos(d*x + c)^3 + (3*A + 4*C)*cos(d*x + c))*sin(d*x + c))/d

Sympy [F]

$$\int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(A+C\sec^2\left(c+dx\right)\right)\cos^4\left(c+dx\right)dx$$

[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2),x)

[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{(dx+c)(3\,A+4\,C)+\frac{(3\,A+4\,C)\tan(dx+c)^3+(5\,A+4\,C)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/8*((d*x + c)*(3*A + 4*C) + ((3*A + 4*C)*tan(d*x + c)^3 + (5*A + 4*C)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{(dx+c)(3\,A+4\,C)+\frac{3\,A\tan(dx+c)^3+4\,C\tan(dx+c)^3+5\,A\tan(dx+c)+4\,C\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/8*((d*x + c)*(3*A + 4*C) + (3*A*tan(d*x + c)^3 + 4*C*tan(d*x + c)^3 + 5*A
*tan(d*x + c) + 4*C*tan(d*x + c))/(tan(d*x + c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = x\left(\frac{3\,A}{8}+\frac{C}{2}\right)+\frac{\left(\frac{3\,A}{8}+\frac{C}{2}\right)\,{\mathrm{tan}\left(c+d\,x\right)}^3+\left(\frac{5\,A}{8}+\frac{C}{2}\right)\,\mathrm{tan}\left(c+d\,x\right)}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^4+2\,{\mathrm{tan}\left(c+d\,x\right)}^2+1\right)}$$

```
[In] int(cos(c + d*x)^4*(A + C/cos(c + d*x)^2),x)
[Out] x*((3*A)/8 + C/2) + (tan(c + d*x)*((5*A)/8 + C/2) + tan(c + d*x)^3*((3*A)/8
 + C/2))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.12 $\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+C)\sin^3(c+dx)}{3d}+\frac{A\sin^5(c+dx)}{5d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(2*A+C)*sin(d*x+c)^3/d+1/5*A*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4129, 3092, 380}

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=-\frac{(2A+C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{A\sin^5(c+dx)}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((2*A + C)*Sin[c + d*x]^3)/(3*d) + (A*Sin[c + d*
x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cos^3(c+dx)\left(C+A\cos^2(c+dx)\right)\,dx\\ &= -\frac{\text{Subst}\left(\int (1-x^2)\left(A+C-Ax^2\right)\,dx, x, -\sin(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(2A+C)x^2+Ax^4\right)\,dx, x, -\sin(c+dx)\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+C)\sin^3(c+dx)}{3d}+\frac{A\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{2A\sin^3(c+dx)}{3d}-\frac{C\sin^3(c+dx)}{3d}+\frac{A\sin^5(c+dx)}{5d}$$

```
[In] Integrate[Cos[c + d*x]^5*(A + C*Sec[c + d*x]^2),x]
[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (2*A*Sin[c + d*x]^3)/(3*d) - (C*S
in[c + d*x]^3)/(3*d) + (A*Sin[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(25A+20C)\sin(3dx+3c)+3A\sin(5dx+5c)+150\sin(dx+c)\left(A+\frac{6C}{5}\right)}{240d}$
derivativedivides	$\frac{\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{5A\sin(dx+c)}{8d}+\frac{3C\sin(dx+c)}{4d}+\frac{A\sin(5dx+5c)}{80d}+\frac{5A\sin(3dx+3c)}{48d}+\frac{\sin(3dx+3c)C}{12d}$
norman	$\frac{-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}-\frac{2(A+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}+\frac{2(A+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{3d}-\frac{4(19A+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{15d}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

```
[In] int(cos(d*x+c)^5*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*((25*A+20*C)*sin(3*d*x+3*c)+3*A*sin(5*d*x+5*c)+150*sin(d*x+c)*(A+6/5*
C))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{\left(3\,A\cos\left(dx+c\right)^4+\left(4\,A+5\,C\right)\cos\left(dx+c\right)^2+8\,A+10\,C\right)\sin\left(dx+c\right)}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*A*cos(d*x + c)^4 + (4*A + 5*C)*cos(d*x + c)^2 + 8*A + 10*C)*sin(d*x
 + c)/d
```

Sympy [F]

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(A+C\sec^2{(c+dx)}\right)\cos^5{(c+dx)}\,dx$$

```
[In] integrate(cos(d*x+c)**5*(A+C*sec(d*x+c)**2),x)
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{3\,A\sin{(dx+c)}^5-5\,(2\,A+C)\sin{(dx+c)}^3+15\,(A+C)\sin{(dx+c)}}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/15*(3*A*sin(d*x + c)^5 - 5*(2*A + C)*sin(d*x + c)^3 + 15*(A + C)*sin(d*x
+ c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{3\,A\sin{(dx+c)}^5-10\,A\sin{(dx+c)}^3-5\,C\sin{(dx+c)}^3+15\,A\sin{(dx+c)}+15\,C\sin{(dx+c)}}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/15*(3*A*sin(d*x + c)^5 - 10*A*sin(d*x + c)^3 - 5*C*sin(d*x + c)^3 + 15*A*
sin(d*x + c) + 15*C*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 15.99 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(c+dx) \left(A+C\sec^2(c+dx)\right) dx = \frac{\frac{A\sin(c+d\,x)^5}{5}+\left(-\frac{2\,A}{3}-\frac{C}{3}\right)\,\sin\left(c+d\,x\right)^3+\left(A+C\right)\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(cos(c + d*x)^5*(A + C/cos(c + d*x)^2),x)
[Out] ((A*sin(c + d*x)^5)/5 + sin(c + d*x)*(A + C) - sin(c + d*x)^3*((2*A)/3 + C/
3))/d
```

3.13 $\int \cos^6(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^6(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{1}{16}(5A+6C)x+\frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{A\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*(5*A+6*C)*x+1/16*(5*A+6*C)*cos(d*x+c)*sin(d*x+c)/d+1/24*(5*A+6*C)*cos(
d*x+c)^3*sin(d*x+c)/d+1/6*A*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4130, 2715, 8}

$$\int \cos^6(c+dx) \left(A+C\sec^2(c+dx)\right) \, dx = \frac{(5A+6C)\sin(c+dx)\cos^3(c+dx)}{24d} + \frac{(5A+6C)\sin(c+dx)\cos(c+dx)}{16d} + \frac{A\sin(c+dx)\cos^5(c+dx)}{6d} + \frac{1}{16}x(5A+6C)$$

```
[In] Int[Cos[c + d*x]^6*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((5*A + 6*C)*x)/16 + ((5*A + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((5*A
 + 6*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5A+6C)\int\cos^4(c+dx)\,dx \\
&= \frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad + \frac{A\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}(5A+6C)\int\cos^2(c+dx)\,dx \\
&= \frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad + \frac{A\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{16}(5A+6C)\int 1\,dx \\
&= \frac{1}{16}(5A+6C)x + \frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad + \frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{A\cos^5(c+dx)\sin(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{60Ac+72cC+60Adx+72Cdx+(45A+48C)\sin(2(c+dx))+(9A+6C)\sin(4(c+dx))+A\sin(6(c+}{192d}$$

[In] Integrate[Cos[c + d*x]^6*(A + C*Sec[c + d*x]^2),x]

[Out] (60*A*c + 72*c*C + 60*A*d*x + 72*C*d*x + (45*A + 48*C)*Sin[2*(c + d*x)] + (
9*A + 6*C)*Sin[4*(c + d*x)] + A*Sin[6*(c + d*x)])/(192*d)

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(45A+48C)\sin(2dx+2c)+(9A+6C)\sin(4dx+4c)+A\sin(6dx+6c)+60dx\left(A+\frac{6C}{5}\right)}{192d}$
risch	$\frac{5Ax}{16}+\frac{3Cx}{8}+\frac{A\sin(6dx+6c)}{192d}+\frac{3A\sin(4dx+4c)}{64d}+\frac{\sin(4dx+4c)C}{32d}+\frac{15A\sin(2dx+2c)}{64d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{\left(-\frac{5A}{16}-\frac{3C}{8}\right)x+\left(-\frac{45A}{16}-\frac{27C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{25A}{16}-\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{25A}{16}-\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{5A}{16}+\frac{3}{8}\right.}{}$

[In] int(cos(d*x+c)^6*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/192*((45*A+48*C)*sin(2*d*x+2*c)+(9*A+6*C)*sin(4*d*x+4*c)+A*sin(6*d*x+6*c)
+60*d*x*(A+6/5*C))/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(c+dx) \left(A+C \sec^2(c+dx)\right) \, dx = \frac{3\,(5\,A+6\,C)dx+\left(8\,A\cos\left(dx+c\right)^5+2\,(5\,A+6\,C)\cos\left(dx+c\right)^3+3\,(5\,A+6\,C)\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{48\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/48*(3*(5*A + 6*C)*d*x + (8*A*cos(d*x + c)^5 + 2*(5*A + 6*C)*cos(d*x + c)^
3 + 3*(5*A + 6*C)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx) \left(A+C \sec^2(c+dx)\right) \, dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(A+C*sec(d*x+c)**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^6(c+dx) \left(A+C \sec^2(c+dx)\right) \, dx = \frac{3\,(dx+c)(5\,A+6\,C)+\frac{3\,(5\,A+6\,C)\tan(dx+c)^5+8\,(5\,A+6\,C)\tan(dx+c)^3+3\,(11\,A+10\,C)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/48*(3*(d*x + c)*(5*A + 6*C) + (3*(5*A + 6*C)*tan(d*x + c)^5 + 8*(5*A + 6*
C)*tan(d*x + c)^3 + 3*(11*A + 10*C)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \cos^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(5\,A+6\,C)+\frac{15\,A\tan(dx+c)^5+18\,C\tan(dx+c)^5+40\,A\tan(dx+c)^3+48\,C\tan(dx+c)^3+33\,A\tan(dx+c)+30\,C\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^3}}{48\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(d*x + c)*(5*A + 6*C) + (15*A*tan(d*x + c)^5 + 18*C*tan(d*x + c)^5
+ 40*A*tan(d*x + c)^3 + 48*C*tan(d*x + c)^3 + 33*A*tan(d*x + c) + 30*C*tan(
d*x + c))/(tan(d*x + c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \cos^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx &= x\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)\\ &\quad+\frac{\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)\,\tan(c+d\,x)^5+\left(\frac{5\,A}{6}+C\right)\,\tan(c+d\,x)^3+\left(\frac{11\,A}{16}+\frac{5\,C}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}\end{aligned}$$

```
[In] int(cos(c + d*x)^6*(A + C/cos(c + d*x)^2),x)
```

```
[Out] x*((5*A)/16 + (3*C)/8) + (tan(c + d*x)*((11*A)/16 + (5*C)/8) + tan(c + d*x)
^3*((5*A)/6 + C) + tan(c + d*x)^5*((5*A)/16 + (3*C)/8))/(d*(3*tan(c + d*x)^
2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.14 $\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 26

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)dx=\frac{C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)}$$

```
[Out] C*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {4128}

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)dx=\frac{C\sin(c+dx)\sec^{m+1}(c+dx)}{d(m+1)}$$

```
[In] Int[Sec[c + d*x]^m*(-((C*m)/(1 + m)) + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m))
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=\frac{C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.35

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)dx$$
$$=\frac{C\csc(c+dx)\sec^{-1+m}(c+dx)\left(-\left((2+m)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\right)+(1+m)\operatorname{Hype}\right.}{d(1+m)(2+m)}$$

```
[In] Integrate[Sec[c + d*x]^m*(-((C*m)/(1 + m)) + C*Sec[c + d*x]^2),x]
[Out] (C*Csc[c + d*x]*Sec[c + d*x]^(-1 + m)*(-((2 + m)*Hypergeometric2F1[1/2, m/2
, (2 + m)/2, Sec[c + d*x]^2]) + (1 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^2)*Sqrt[-Tan[c + d*x]^2])/(d*(1 + m)
*(2 + m))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

method	result
parallelrisch	$-\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\frac{1}{\cos(dx+c)}\right)^m}{(1+m)d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$-\frac{iC\left(e^{i(dx+c)}\right)^m\left(e^{2i(dx+c)}+1\right)^{-m}2^m\left(e^{\frac{i\pi\operatorname{csgn}\left(\frac{i}{e^{2i(dx+c)}+1}\right)\operatorname{csgn}\left(\frac{ie^{i(dx+c)}}{e^{2i(dx+c)}+1}\right)^2m}{2}}e^{-\frac{i\pi\operatorname{csgn}\left(\frac{i}{e^{2i(dx+c)}+1}\right)\operatorname{csgn}\left(\frac{ie^{i(dx+c)}}{e^{2i(dx+c)}+\right.}{2}}\right.}{}$

```
[In] int(sec(d*x+c)^m*(-C*m/(1+m)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -2*C/(1+m)/d*tan(1/2*d*x+1/2*c)*(1/cos(d*x+c))^m/(tan(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)dx=\frac{C\frac{1}{\cos(dx+c)}^m\sin(dx+c)}{(dm+d)\cos(dx+c)}$$

```
[In] integrate(sec(d*x+c)^m*(-C*m/(1+m)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] C*(1/cos(d*x + c))^m*sin(d*x + c)/((d*m + d)*cos(d*x + c))
```

Sympy [F]

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{C\left(\int\left(-m\sec^m\left(c+dx\right)\right)\,dx+\int\sec^2\left(c+dx\right)\sec^m\left(c+dx\right)dx+\int m\sec^2\left(c+dx\right)\sec^m\left(c+dx\right)dx\right)}{m+1}$$

```
[In] integrate(sec(d*x+c)**m*(-C*m/(1+m)+C*sec(d*x+c)**2),x)
```

```
[Out] C*(Integral(-m*sec(c + d*x)**m, x) + Integral(sec(c + d*x)**2*sec(c + d*x)*
*m, x) + Integral(m*sec(c + d*x)**2*sec(c + d*x)**m, x))/(m + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(26) = 52$.

Time = 0.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 11.69

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)\,dx =$$
$$-\frac{2^mC\cos\left(-(dx+c)(m+2)+m\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)\sin\left(2\,dx+2\,c\right)-2^mC\,c}{}$$

```
[In] integrate(sec(d*x+c)^m*(-C*m/(1+m)+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(2^m*C*cos(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*sin(2*d*x + 2*c) - 2^m*C*cos(-(d*x + c)*m + m*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) + (2^m*C*cos(2*d*x + 2*c) + 2
^m*C)*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - (2^m*C*cos(2*d*x + 2*c) + 2^m*C)*sin(-(d*x + c)*m + m*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((m + 1)*cos(2*d*x + 2*c)^2 + (m + 1
)*sin(2*d*x + 2*c)^2 + 2*(m + 1)*cos(2*d*x + 2*c) + m + 1)*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*d)
```

Giac [F]

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)\,dx$$
$$=\int\left(C\sec\left(dx+c\right)^2-\frac{Cm}{m+1}\right)\sec\left(dx+c\right)^m\,dx$$

```
[In] integrate(sec(d*x+c)^m*(-C*m/(1+m)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 - C*m/(m + 1))*sec(d*x + c)^m, x)
```

Mupad [B] (verification not implemented)

Time = 15.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \sec^m(c+dx)\left(-\frac{Cm}{1+m}+C\sec^2(c+dx)\right)\,dx=\frac{C\,\sin\left(2\,c+2\,d\,x\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^m}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\left(m+1\right)}$$

```
[In] int((1/cos(c + d*x))^m*(C/cos(c + d*x)^2 - (C*m)/(m + 1)),x)
[Out] (C*sin(2*c + 2*d*x)*(1/cos(c + d*x))^m)/(d*(cos(2*c + 2*d*x) + 1)*(m + 1))
```

3.15 $\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 25

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx=-\frac{A\sec^{1+m}(c+dx)\sin(c+dx)}{dm}$$

```
[Out] -A*sec(d*x+c)^(1+m)*sin(d*x+c)/d/m
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {4128}

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx=-\frac{A\sin(c+dx)\sec^{m+1}(c+dx)}{dm}$$

```
[In] Int[Sec[c + d*x]^m*(A - (A*(1 + m)*Sec[c + d*x]^2)/m),x]
```

```
[Out] -((A*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*m))
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{A\sec^{1+m}(c+dx)\sin(c+dx)}{dm}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.44

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx$$
$$=\frac{A\csc(c+dx)\sec^{-1+m}(c+dx)\left((2+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)-(1+m)\,\text{Hyperge}\right.}{dm(2+m)}$$

```
[In] Integrate[Sec[c + d*x]^m*(A - (A*(1 + m)*Sec[c + d*x]^2)/m),x]
```

```
[Out] (A*Csc[c + d*x]*Sec[c + d*x]^(-1 + m)*((2 + m)*Hypergeometric2F1[1/2, m/2,
(2 + m)/2, Sec[c + d*x]^2] - (1 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^2)*Sqrt[-Tan[c + d*x]^2])/(d*m*(2 + m))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result
parallelrisch	$\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\frac{1}{\cos(dx+c)}\right)^m}{md\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{iA\left(e^{i(dx+c)}\right)^m\left(e^{2i(dx+c)}+1\right)^{-m}2^m\left(e^{\frac{i\pi\,\text{csgn}\left(\frac{i}{e^{2i(dx+c)}+1}\right)\text{csgn}\left(\frac{ie^{i(dx+c)}}{e^{2i(dx+c)}+1}\right)^2 m}{2}}e^{-\frac{i\pi\,\text{csgn}\left(\frac{i}{e^{2i(dx+c)}+1}\right)\text{csgn}\left(\frac{ie^{i(dx+c)}}{e^{2i(dx+c)}+1}\right.}{2}}\right.}{}$

```
[In] int(sec(d*x+c)^m*(A-A*(1+m)*sec(d*x+c)^2/m),x,method=_RETURNVERBOSE)
```

```
[Out] 2*A/m/d*tan(1/2*d*x+1/2*c)*(1/cos(d*x+c))^m/(tan(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx=-\frac{A\frac{1}{\cos(dx+c)}^m\sin\left(dx+c\right)}{dm\cos\left(dx+c\right)}$$

```
[In] integrate(sec(d*x+c)^m*(A-A*(1+m)*sec(d*x+c)^2/m),x, algorithm="fricas")
```

```
[Out] -A*(1/cos(d*x + c))^m*sin(d*x + c)/(d*m*cos(d*x + c))
```

Sympy [F]

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)dx = -\frac{A\left(\int(-m\sec^m(c+dx))\,dx+\int\sec^2(c+dx)\sec^m(c+dx)\,dx+\int m\sec^2(c+dx)\sec^m(c+dx)\,dx\right)}{m}$$

```
[In] integrate(sec(d*x+c)**m*(A-A*(1+m)*sec(d*x+c)**2/m),x)
[Out] -A*(Integral(-m*sec(c + d*x)**m, x) + Integral(sec(c + d*x)**2*sec(c + d*x)
**m, x) + Integral(m*sec(c + d*x)**2*sec(c + d*x)**m, x))/m
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(25) = 50$.

Time = 0.43 (sec) , antiderivative size = 296, normalized size of antiderivative = 11.84

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)dx = \frac{2^mA\cos(-(dx+c)(m+2)+m\arctan(\sin(2dx+2c),\cos(2dx+2c)+1))\sin(2dx+2c)-2^mA\cos}{}$$

```
[In] integrate(sec(d*x+c)^m*(A-A*(1+m)*sec(d*x+c)^2/m),x, algorithm="maxima")
[Out] (2^m*A*cos(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*sin(2*d*x + 2*c) - 2^m*A*cos(-(d*x + c)*m + m*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) + (2^m*A*cos(2*d*x + 2*c) + 2^
m*A)*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - (2^m*A*cos(2*d*x + 2*c) + 2^m*A)*sin(-(d*x + c)*m + m*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((m*cos(2*d*x + 2*c)^2 + m*sin(2*d*x +
 2*c)^2 + 2*m*cos(2*d*x + 2*c) + m)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*d)
```

Giac [F]

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)dx = \int -\left(\frac{A(m+1)\sec(dx+c)^2}{m}-A\right)\sec(dx+c)^m\,dx$$

```
[In] integrate(sec(d*x+c)^m*(A-A*(1+m)*sec(d*x+c)^2/m),x, algorithm="giac")
[Out] integrate(-(A*(m + 1)*sec(d*x + c)^2/m - A)*sec(d*x + c)^m, x)
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \sec^m(c+dx)\left(A-\frac{A(1+m)\sec^2(c+dx)}{m}\right)\,dx=-\frac{A\,\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^m}{d\,m\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int((A - (A*(m + 1))/(m*cos(c + d*x)^2))*(1/cos(c + d*x))^m,x)
[Out] -(A*sin(2*c + 2*d*x)*(1/cos(c + d*x))^m)/(d*m*(cos(2*c + 2*d*x) + 1))
```

3.16 $\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b(7A+5C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2C(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

```
[Out] 2/21*b*(7*A+5*C)*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/21*b^2*(7*A+5*C)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/7*C*(b*sec(d*x+c))^(5/2)*ta
n(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4131, 3853, 3856, 2720}

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b(7A+5C)\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}+\frac{2C\tan(c+dx)(b\sec(c+dx))^{5/2}}{7d}$$

```
[In] Int[(b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(21*d) + (2*b*(7*A + 5*C)*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
21*d) + (2*C*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{1}{7}(7A+5C)\int(b\sec(c+dx))^{5/2}\,dx\\
&= \frac{2b(7A+5C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2C(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}\\
&\quad+\frac{1}{21}\left(b^2(7A+5C)\right)\int\sqrt{b\sec(c+dx)}\,dx\\
&= \frac{2b(7A+5C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2C(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}\\
&\quad+\frac{1}{21}\left(b^2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}\\
&\quad+\frac{2b(7A+5C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2C(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{(b\sec(c+dx))^{7/2}\left(4(7A+5C)\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2(7A+11C+$$
$$42bd$$

[In] Integrate[(b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]

[Out] ((b*Sec[c + d*x])^(7/2)*(4*(7*A + 5*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(7*A + 11*C + (7*A + 5*C)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*b*d)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 22.69 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.65

method	result
default	$\frac{2b^2\sqrt{b\sec(dx+c)}\left(7iA\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\cos(dx+c)+5iC\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+}}\right.}{}$
parts	$\frac{2A\sqrt{b\sec(dx+c)}\,b^2\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\cos(dx+c)+i\,\text{EllipticF}(i(\cot(dx+c)-\csc(\right.}{3d}$

[In] int((b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 2/21*b^2/d*(b*sec(d*x+c))^(1/2)*(7*I*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)+5*I*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)+7*I*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+5*I*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+7*A*tan(d*x+c)+5*C*tan(d*x+c)+3*C*tan(d*x+c)*sec(d*x+c)^2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{-i\sqrt{2}(7A+5C)b^{\frac{5}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+$$

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(5/2)*cos(d*x
+ c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((7*A
+ 5*C)*b^2*cos(d*x + c)^2 + 3*C*b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^3)
```

Sympy [F]

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \int (b\sec(c+dx))^{\frac{5}{2}}\left(A+C\sec^2(c+dx)\right)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
[Out] Integral((b*sec(c + d*x))**(5/2)*(A + C*sec(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{5/2} \left(A+C\sec^2(c+dx)\right) \, dx = \int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}} \, dx$$

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{5/2} \left(A+C\sec^2(c+dx)\right) \, dx = \int \left(A+\frac{C}{\cos\left(c+d\,x\right)^2}\right)\,\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2} \,dx$$

```
[In] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2),x)
[Out] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2), x)
```

3.17 $\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=-\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] -2/5*b^2*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b
*(5*A+3*C)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/5*C*(b*sec(d*x+c))^(3/2)*tan
(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4131, 3853, 3856, 2719}

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=-\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b(5A+3C)\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2C\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*b^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*b*(5*A + 3*C)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*
d) + (2*C*(b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{1}{5}(5A+3C)\int (b\sec(c+dx))^{3/2}\,dx\\
&= \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}\\
&\quad -\frac{1}{5}\left(b^2(5A+3C)\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}\\
&\quad +\frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}-\frac{\left(b^2(5A+3C)\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= -\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}\\
&\quad +\frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=\frac{4ie^{i(c+dx)}\cos^3(c+dx)\left(-3\left(5A\left(1+e^{2i(c+dx)}\right)^2+C\left(1+8e^{2i(c+dx)}+3e^{4i(c+dx)}\right)\right)+(5A}{15d\left(1+e^{2i(c+}\right.}$$

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (((4*I)/15)*E^(I*(c + d*x))*Cos[c + d*x]^3*(-3*(5*A*(1 + E^((2*I)*(c + d*x)
))^2 + C*(1 + 8*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x)))) + (5*A + 3*C)
*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)
*(c + d*x))])*(b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/(d*(1 + E^((2*
I)*(c + d*x)))^2*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.33 (sec) , antiderivative size = 798, normalized size of antiderivative = 7.25

method	result	size
default	Expression too large to display	798
parts	Expression too large to display	809

```
[In] int((b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*b/d*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(-3*I*C*EllipticE(I*(cot(d*x+c
)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)^2+6*I*C*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+5*I*A*(1/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x
+c)),I)+5*I*A*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+3*I*C*EllipticF(I*(cot(d*
x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)^2-10*I*A*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+10*I*A*EllipticF(
I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)+3*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-5*I*A*EllipticE(I*(cot
(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
```

```
^(1/2)*cos(d*x+c)^2-5*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-6*I*C*EllipticE(I*(cot(d*x
+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)-3*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-5*A*sin(d*x+c)-3*C*sin(d*x+c)-C*
tan(d*x+c)-C*sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{-i\sqrt{2}(5A+3C)b^{\frac{3}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,c$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(5*A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(5*A
+ 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((5*A + 3*C)*b*cos(d*x + c)^2 + C*
b)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\left(A+C\sec^2(c+dx)\right)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*(A + C*sec(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx=\int \left(A+\frac{C}{\cos\left(c+d\,x\right)^2}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2),x)
[Out] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b\sec(c+dx)}(A+C\sec^2(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/3*C*(b*sec
(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4131, 3856, 2720}

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2C\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}$$

```
[In] Int[Sqrt[b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*
x]])/(3*d) + (2*C*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}(3A+C)\int\sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{1}{3}\left((3A+C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int\sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{2(b\sec(c+dx))^{3/2}\left((3A+C)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(c+dx)\right)}{3bd}$$

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(b*Sec[c + d*x])^(3/2)*((3*A + C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)
/2, 2] + C*Sin[c + d*x]))/(3*b*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.06

method	result
parts	$-\frac{2iA(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{b\sec(dx+c)}}{d}-\frac{2C\sqrt{b\sec(dx+c)}\left(i\sqrt{\right.}}{}$
default	$\frac{2\sqrt{b\sec(dx+c)}\left(3iA\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\cos(dx+c)+iC\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+}}\right.}{}$

```
[In] int((b*sec(d*x+c))^(1/2)*(A+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*A/d*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(b*sec(d*x+c))^(1/2)-2/3*C/
d*(b*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)+I*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc
(d*x+c)),I)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}(3i\,A+i\,C}{3\,d\cos\left(d\right.}$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*cos(d*x + c
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*C*sqrt(b/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \int \sqrt{b\sec\left(c+dx\right)}\left(A+C\sec^2\left(c+dx\right)\right)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(1/2)*(A+C*sec(d*x+c)**2),x)
[Out] Integral(sqrt(b*sec(c + d*x))*(A + C*sec(c + d*x)**2), x)
```

Maxima [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \int \left(A+\frac{C}{\cos\left(c+d\,x\right)^2}\right)\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2),x)
[Out] int((A + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2), x)
```

3.19 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*C*tan(d*x+c)/d
/(b*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4131, 3856, 2719}

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(A + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*C*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}} + (A-C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}} + \frac{(A-C)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.79 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$
\int \frac{A+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = -\frac{2i\left(-3\left(A+Ae^{2i(c+dx)}-2Ce^{2i(c+dx)}\right)+2(A-C)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2}\right.}{3d\left(1+e^{2i(c+dx)}\right)\sqrt{b\sec(c+dx)}}
$$

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (((-2*I)/3)*(-3*(A + A*E^((2*I)*(c + d*x)) - 2*C*E^((2*I)*(c + d*x))) + 2*(
A - C)*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*(1 + E^((2*I)*(c + d*x)))*Sqrt[b*
Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 763, normalized size of antiderivative = 11.22

method	result	size
default	Expression too large to display	763
parts	Expression too large to display	778

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*A*EllipticF(I*(cot(d*x+c)-csc(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)-I*A*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*C*EllipticF(I*(cot(d*x+c)-csc(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)+I*C*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+2*I*A*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-2*I
*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cot(d*x+c)-csc(d*x+c)),I)-2*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+2*I*C*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*
x+c)),I)+I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)-I*A*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*s
ec(d*x+c)-I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)+I*C*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*
sec(d*x+c)+A*sin(d*x+c)+C*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \frac{\sqrt{2}(i\,A - i\,C)\sqrt{b}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\,\sin(dx + c))) + \sqrt{2}(-}{}$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(I*A - I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*A + I*C)*sqrt(b)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*C*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)
[Out] Integral((A + C*sec(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\sec(dx + c)^2 + A}{\sqrt{b\sec(dx + c)}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\sec(dx + c)^2 + A}{\sqrt{b\sec(dx + c)}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{A + \frac{C}{\cos(c+d\,x)^2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
[Out] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

3.20 $\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d+2/3*A*ta
n(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4130, 3856, 2720}

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*
x]])/(3*b^2*d) + (2*A*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}} + \frac{(A+3C)\int\sqrt{b\sec(c+dx)}\,dx}{3b^2} \\
&= \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2} \\
&= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int\frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^2(c+dx)\left(2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin(2(c+dx\right.}{3d(b\sec(c+dx))^{3/2}}$$

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
[Out] (Sec[c + d*x]^2*(2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 A*Sin[2*(c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.91

method	result
parts	$-\frac{2A\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx\right.}{3d\sqrt{b\sec(dx+c)}\,b}$
default	$\frac{\frac{2iA\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{3}+2iC\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx}{}$

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/d/(b*sec(d*x+c))^(1/2)/b*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d
*x+c)),I)*sec(d*x+c)-sin(d*x+c))-2*I*C/d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-cot(d*x+c)+csc(d*x+c)),I)/b/(b*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\,A\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+\sqrt{2}(-i\,A-3i\,C)\sqrt{b}\text{weierstrassPInver}}{}$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-I*A - 3
*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + s
qrt(2)*(I*A + 3*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)))/(b^2*d)
```

Sympy [F]

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{A+C\sec^2{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
[Out] Integral((A + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\sec{(dx+c)}^2+A}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\sec{(dx+c)}^2+A}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{A+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
[Out] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

3.21 $\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[Out] 2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*A*
tan(d*x+c)/d/(b*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4130, 3856, 2719}

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[
b*Sec[c + d*x]]) + (2*A*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}+\frac{(3A+5C)\int\frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}\\ &= \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}+\frac{(3A+5C)\int\sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.73

$$\int\frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{e^{-idx}\sec^2(c+dx)(\cos(dx)+i\sin(dx))\left(12i(3A+5C)-\frac{8i(3A+5C)e^{2i(c+dx)}\text{Hypergeon}}{\sqrt{1+e^{2i}}}\right.}{30d(b\sec(c+dx))^{5/2}}$$

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])*((12*I)*(3*A + 5*C) - ((8*I)*(3*A +
 5*C)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 6*A*Sin[2*(c + d*x)]))/(30*d*E^(I*d*
x)*(b*Sec[c + d*x])^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.27 (sec) , antiderivative size = 798, normalized size of antiderivative = 10.36

method	result	size
default	Expression too large to display	798
parts	Expression too large to display	812

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/b^2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(-3*I*A*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I
)*sec(d*x+c)-3*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)+6*I*A*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+
c)),I)-10*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+5*I*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c
)+10*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(cot(d*x+c)-csc(d*x+c)),I)+3*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)-6*I
*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cot(d*x+c)-csc(d*x+c)),I)-5*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)-5*I*C*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*
x+c)-csc(d*x+c)),I)*cos(d*x+c)+A*cos(d*x+c)^2*sin(d*x+c)+5*I*C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(
d*x+c)),I)*cos(d*x+c)+3*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)+A*cos(d*x+c)*
sin(d*x+c)+3*A*sin(d*x+c)+5*C*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \frac{2\,A\sqrt{\frac{b}{\cos(dx+c)}}\cos{(dx + c)}^2\sin{(dx + c)} + \sqrt{2}(3i\,A + 5i\,C)\sqrt{b}\text{weierstrassZeta}(-$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*A*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + sqrt(2)*(3*I*A
+ 5*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
```

```
+ c) + I*sin(d*x + c))) + sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{A + C\sec^2\left(c + dx\right)}{\left(b\sec\left(c + dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
[Out] Integral((A + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{C\sec\left(dx + c\right)^2 + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{C\sec\left(dx + c\right)^2 + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{5/2}}\,dx = \int \frac{A + \frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
[Out] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

3.22 $\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[Out] 2/21*(5*A+7*C)*sin(d*x+c)/b^3/d/(b*sec(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^4/d+2/7*A*tan(d*x+c)/d/(b*sec(d
*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4130, 3854, 3856, 2720}

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(21*b^4*d) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*Sqrt[b*Sec[c + d
*x]]) + (2*A*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}+\frac{(5A+7C)\int\frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b^2}\\
&= \frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}+\frac{(5A+7C)\int\sqrt{b\sec(c+dx)}\,dx}{21b^4}\\
&= \frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\\
&\quad+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^4}\\
&= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d}\\
&\quad+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\frac{\frac{4(5A+7C)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}}+2(13A+14C+3A\cos(2(c+dx)))\sin(c+dx)}{42b^3d\sqrt{b\sec(c+dx)}}$$

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((4*(5*A + 7*C)*EllipticF[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(13*A + 1
4*C + 3*A*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*b^3*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.66 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.60

method	result
default	$\frac{\frac{10iA\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{21}+\frac{2iC\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)}{3}}{}$
parts	$-\frac{2A\left(5i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+5i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(-\cot(c\right.}{21d\sqrt{b\sec(dx+c)}\,b^3}$

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b^3/d/(b*sec(d*x+c))^(1/2)*(5*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+7*I*C*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)
-csc(d*x+c)),I)+5*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)+3*A*cos(d*x+c)^2*si
n(d*x+c)+7*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)+5*A*sin(d*x+c)+7*C*sin(d*x
+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,A-7i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+}{}$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*A*cos(d*x + c)^3 + (5*A + 7*C
)*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^4*d)
```

Sympy [F]

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{A+C\sec^2{(c+dx)}}{(b\sec{(c+dx)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(7/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{C\sec{(dx+c)}^2+A}{(b\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{7/2}}\,dx = \int \frac{C\sec\left(dx + c\right)^2 + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{7/2}}\,dx = \int \frac{A + \frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2),x)
[Out] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2), x)
```

3.23 $\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(7A+9C)\sin(c+dx)}{45b^3d(b\sec(c+dx))^{3/2}} + \frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

```
[Out] 2/45*(7*A+9*C)*sin(d*x+c)/b^3/d/(b*sec(d*x+c))^(3/2)+2/15*(7*A+9*C)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/b^4/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/9*A*tan(d*x+c)/d/(b*sec(d
*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4130, 3854, 3856, 2719}

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(7A+9C)\sin(c+dx)}{45b^3d(b\sec(c+dx))^{3/2}} + \frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*b^4*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*(7*A + 9*C)*Sin[c + d*x])/(45*b^3*d*(b*Sec[c + d*x])
^(3/2)) + (2*A*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{(7A+9C)\int\frac{1}{(b\sec(c+dx))^{5/2}}\,dx}{9b^2}\\
&= \frac{2(7A+9C)\sin(c+dx)}{45b^3d(b\sec(c+dx))^{3/2}}+\frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{(7A+9C)\int\frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b^4}\\
&= \frac{2(7A+9C)\sin(c+dx)}{45b^3d(b\sec(c+dx))^{3/2}}+\frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{(7A+9C)\int\sqrt{\cos(c+dx)}\,dx}{15b^4\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2(7A+9C)\sin(c+dx)}{45b^3d(b\sec(c+dx))^{3/2}}+\frac{2A\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \frac{e^{-idx}(\cos(dx)+i\sin(dx))\left(336iA+432iC-\frac{32i(7A+9C)e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.}{\sqrt{1+e^{2i(c+dx)}}}\right.}{360b^4d\sqrt{b\sec(c+}}$$

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*((336*I)*A + (432*I)*C - ((32*I)*(7*A + 9*C)*E^((2
*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt
[1 + E^((2*I)*(c + d*x))] + (76*A + 72*C)*Sin[2*(c + d*x)] + 10*A*Sin[4*(c
+ d*x)]))/(360*b^4*d*E^(I*d*x)*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.76 (sec) , antiderivative size = 866, normalized size of antiderivative = 7.73

method	result	size
default	Expression too large to display	866
parts	Expression too large to display	876

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/b^4/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(-27*I*C*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)
),I)*sec(d*x+c)+42*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-5*A*cos(d*x+c)^4*sin(d*x+c)-2
1*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(
I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)+21*I*A*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d
*x+c)+54*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-42*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-5*A*cos(d
*x+c)^3*sin(d*x+c)+27*I*C*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*sec(d*x+c)-54*I*C*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-
csc(d*x+c)),I)-21*I*A*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-27*I*C*EllipticF(I*
(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)-7*A*cos(d*x+c)^2*sin(d*x+c)+21*I*A*EllipticE(I*(cot(d
```

```
*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)+27*I*C*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-9*C*cos(d*x+c)^2*s
in(d*x+c)-7*A*cos(d*x+c)*sin(d*x+c)-9*C*cos(d*x+c)*sin(d*x+c)-21*A*sin(d*x+
c)-27*C*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

$$\int \frac{A + C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-7i\,A - 9i\,C)\sqrt{b}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))) +}{}$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-7*I*A - 9*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(7*I*A + 9*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*A*cos(d*x + c)^4 + (7*A + 9*C)*cos(d*x + c)^2)*sqrt
(b/cos(d*x + c))*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \int \frac{C\sec(dx+c)^2 + A}{(b\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{9/2}}\,dx = \int \frac{C\sec{(dx + c)}^2 + A}{(b\sec{(dx + c)})^{\frac{9}{2}}}\,dx$$

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sec^2(c + dx)}{(b\sec(c + dx))^{9/2}}\,dx = \int \frac{A + \frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

```
[In] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(9/2),x)
[Out] int((A + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(9/2), x)
```

3.24 $\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\frac{6\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] 6*sin(d*x+c)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {4128}

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

```
[In] Int[(3 + 3*Sec[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
[Out] (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=\frac{6\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{6\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

```
[In] Integrate[(3 + 3*Sec[c + d*x]^2)/Sqrt[Sec[c + d*x]],x]
[Out] (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 1.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

method	result
default	$\frac{12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$\frac{6\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d} - 6\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.$

```
[In] int((3+3*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 12*sin(1/2*d*x+1/2*c)*cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{6\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

```
[In] integrate((3+3*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 6*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F]

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = 3\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

```
[In] integrate((3+3*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
[Out] 3*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{3\left(\sec(dx+c)^2+1\right)}{\sqrt{\sec(dx+c)}}\,dx$$

```
[In] integrate((3+3*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 3*integrate((sec(d*x + c)^2 + 1)/sqrt(sec(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = -\frac{12\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)d}$$

```
[In] integrate((3+3*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] -12*sqrt(-tan(1/2*d*x + 1/2*c)^4 + 1)*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^4 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{3+3\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{6\sin(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{d}$$

```
[In] int((3/cos(c + d*x)^2 + 3)/(1/cos(c + d*x))^(1/2),x)
[Out] (6*sin(c + d*x)*(1/cos(c + d*x))^(1/2))/d
```

3.25 $\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 21

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx=-\frac{\sec^{1+m}(e+fx)\sin(e+fx)}{f}$$

```
[Out] -sec(f*x+e)^(1+m)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {4128}

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)\sec^{m+1}(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]^m*(m - (1 + m)*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^(1 + m)*Sin[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\sec^{1+m}(e+fx)\sin(e+fx)}{f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 5.10

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx$$
$$=\frac{\csc(e+fx)\sec^{-1+m}(e+fx)\left((2+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)-(1+m)\,\text{Hyperge}\ldots\right.}{f(2+m)}$$

```
[In] Integrate[Sec[e + f*x]^m*(m - (1 + m)*Sec[e + f*x]^2),x]
```

```
[Out] (Csc[e + f*x]*Sec[e + f*x]^(-1 + m)*((2 + m)*Hypergeometric2F1[1/2, m/2, (2
 + m)/2, Sec[e + f*x]^2] - (1 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, Sec[e + f*x]^2]*Sec[e + f*x]^2)*Sqrt[-Tan[e + f*x]^2])/(f*(2 + m))
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result
parallelrisch	$\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\frac{1}{\cos(fx+e)}\right)^m}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$
risch	$\frac{i\left(e^{i(fx+e)}\right)^m\left(e^{2i(fx+e)}+1\right)^{-m}2^m\left(e^{\frac{i\,\text{csgn}\left(\frac{i}{e^{2i(fx+e)}+1}\right)\text{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2\pi m}{2}}e^{-\frac{i\,\text{csgn}\left(\frac{i}{e^{2i(fx+e)}+1}\right)\text{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)\ldots}{2}}\ldots\right.}{\ldots}$

```
[In] int(sec(f*x+e)^m*(m-(1+m)*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*tan(1/2*f*x+1/2*e)*(1/cos(f*x+e))^m/(tan(1/2*f*x+1/2*e)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx=-\frac{\frac{1}{\cos(fx+e)}^m\sin(fx+e)}{f\cos(fx+e)}$$

```
[In] integrate(sec(f*x+e)^m*(m-(1+m)*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -(1/cos(f*x + e))^m*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx$$
$$= -\int \left(-m\sec^m\left(e+fx\right)\right)\,dx - \int \sec^2\left(e+fx\right)\sec^m\left(e+fx\right)dx$$
$$- \int m\sec^2\left(e+fx\right)\sec^m\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**m*(m-(1+m)*sec(f*x+e)**2),x)
```

```
[Out] -Integral(-m*sec(e + f*x)**m, x) - Integral(sec(e + f*x)**2*sec(e + f*x)**m
, x) - Integral(m*sec(e + f*x)**2*sec(e + f*x)**m, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(21) = 42$.

Time = 0.43 (sec) , antiderivative size = 283, normalized size of antiderivative = 13.48

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx$$
$$= \frac{2^m\cos\left(-(fx+e)(m+2)+m\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)\sin\left(2\,fx+2\,e\right)-2^m\cos\left(-(\ldots\right.}{}$$

```
[In] integrate(sec(f*x+e)^m*(m-(1+m)*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (2^m*cos(-(f*x + e)*(m + 2) + m*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1))*sin(2*f*x + 2*e) - 2^m*cos(-(f*x + e)*m + m*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1))*sin(2*f*x + 2*e) + (2^m*cos(2*f*x + 2*e) + 2^m)*sin
(-(f*x + e)*(m + 2) + m*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
(2^m*cos(2*f*x + 2*e) + 2^m)*sin(-(f*x + e)*m + m*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/2*m)*f)
```

Giac [F]

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx$$
$$=\int -\left((m+1)\sec\left(fx+e\right)^2-m\right)\sec\left(fx+e\right)^m\,dx$$

```
[In] integrate(sec(f*x+e)^m*(m-(1+m)*sec(f*x+e)^2),x, algorithm="giac")
[Out] integrate(-((m + 1)*sec(f*x + e)^2 - m)*sec(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \sec^m(e+fx)\left(m-(1+m)\sec^2(e+fx)\right)\,dx=-\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^m}{f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

```
[In] int((m - (m + 1)/cos(e + f*x)^2)*(1/cos(e + f*x))^m,x)
[Out] -(sin(2*e + 2*f*x)*(1/cos(e + f*x))^m)/(f*(cos(2*e + 2*f*x) + 1))
```

3.26 $\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx=-\frac{\sec^5(e+fx)\tan(e+fx)}{f}$$

```
[Out] -sec(f*x+e)^5*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx=-\frac{\tan(e+fx)\sec^5(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]^5*(5 - 6*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^5*Tan[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\sec^5(e+fx)\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx=-\frac{\sec^5(e+fx)\tan(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]^5*(5 - 6*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^5*Tan[e + f*x])/f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

method	result
risch	$\frac{32i\left(\mathrm{e}^{7i(fx+e)}-\mathrm{e}^{5i(fx+e)}\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{6}}$
parallelrisch	$-\frac{32\sin(fx+e)}{f(\cos(6fx+6e)+6\cos(4fx+4e)+15\cos(2fx+2e)+10)}$
derivativedivides	$\frac{-5\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+6\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)}{f}$
default	$\frac{-5\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+6\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)}{f}$
parts	$\frac{-5\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{15\ln(\sec(fx+e)+\tan(fx+e))}{8}}{f}-\frac{6\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx}{16}\right.\right.}{}$
norman	$\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}-\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$

```
[In] int(sec(f*x+e)^5*(5-6*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 32*I/f/(exp(2*I*(f*x+e))+1)^6*(exp(7*I*(f*x+e))-exp(5*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^5(e+fx) \left(5-6\sec^2(e+fx)\right) \, dx = -\frac{\sin\left(fx+e\right)}{f\cos\left(fx+e\right)^6}$$

```
[In] integrate(sec(f*x+e)^5*(5-6*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec^5(e+fx) \left(5-6\sec^2(e+fx)\right) \, dx = -\int \left(-5\sec^5\left(e+fx\right)\right) \, dx - \int 6\sec^7\left(e+fx\right) dx$$

```
[In] integrate(sec(f*x+e)**5*(5-6*sec(f*x+e)**2),x)
[Out] -Integral(-5*sec(e + f*x)**5, x) - Integral(6*sec(e + f*x)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \sec^5(e+fx) \left(5-6\sec^2(e+fx)\right) \, dx = \frac{\sin\left(fx+e\right)}{\left(\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx+e\right)^2-1\right)f}$$

```
[In] integrate(sec(f*x+e)^5*(5-6*sec(f*x+e)^2),x, algorithm="maxima")
[Out] sin(f*x + e)/((sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx=\frac{\sin(fx+e)}{\left(\sin(fx+e)^2-1\right)^3f}$$

```
[In] integrate(sec(f*x+e)^5*(5-6*sec(f*x+e)^2),x, algorithm="giac")
[Out] sin(f*x + e)/((sin(f*x + e)^2 - 1)^3*f)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \sec^5(e+fx)\left(5-6\sec^2(e+fx)\right)\,dx$$
$$=\frac{\sin(e+f\,x)}{f\,\left(\sin(e+f\,x)^6-3\sin(e+f\,x)^4+3\sin(e+f\,x)^2-1\right)}$$

```
[In] int(-(6/cos(e + f*x)^2 - 5)/cos(e + f*x)^5,x)
[Out] sin(e + f*x)/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.27 $\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\sec^4(e+fx)\tan(e+fx)}{f}$$

```
[Out] -sec(f*x+e)^4*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\tan(e+fx)\sec^4(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]^4*(4 - 5*Sec[e + f*x]^2),x]
```

```
[Out] -((Sec[e + f*x]^4*Tan[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\sec^4(e+fx)\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\sec^4(e+fx)\tan(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]^4*(4 - 5*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^4*Tan[e + f*x])/f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

method	result	size
risch	$\frac{16i\left(\mathrm{e}^{6i(fx+e)}-\mathrm{e}^{4i(fx+e)}\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{5}}$	41
parallelrisch	$-\frac{16\sin(fx+e)}{f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$	43
derivativedivides	$\frac{-4\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)+5\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)}{f}$	56
default	$\frac{-4\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)+5\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)}{f}$	56
parts	$-\frac{4\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}+\frac{5\left(-\frac{8}{15}-\frac{\sec(fx+e)^{4}}{5}-\frac{4\sec(fx+e)^{2}}{15}\right)\tan(fx+e)}{f}$	58
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f}+\frac{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{f}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{5}}$	96

```
[In] int(sec(f*x+e)^4*(4-5*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 16*I/f/(exp(2*I*(f*x+e))+1)^5*(exp(6*I*(f*x+e))-exp(4*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\sin\left(fx+e\right)}{f\cos\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)^4*(4-5*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\int\left(-4\sec^4\left(e+fx\right)\right)\,dx-\int 5\sec^6\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**4*(4-5*sec(f*x+e)**2),x)
[Out] -Integral(-4*sec(e + f*x)**4, x) - Integral(5*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\tan\left(fx+e\right)^5+2\tan\left(fx+e\right)^3+\tan\left(fx+e\right)}{f}$$

```
[In] integrate(sec(f*x+e)^4*(4-5*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(tan(f*x + e)^5 + 2*tan(f*x + e)^3 + tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\tan\left(fx+e\right)^5+2\tan\left(fx+e\right)^3+\tan\left(fx+e\right)}{f}$$

```
[In] integrate(sec(f*x+e)^4*(4-5*sec(f*x+e)^2),x, algorithm="giac")
[Out] -(tan(f*x + e)^5 + 2*tan(f*x + e)^3 + tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^4(e+fx)\left(4-5\sec^2(e+fx)\right)\,dx=-\frac{\sin\left(e+f\,x\right)}{f\,{\cos\left(e+f\,x\right)}^5}$$

```
[In] int(-(5/cos(e + f*x)^2 - 4)/cos(e + f*x)^4,x)
[Out] -sin(e + f*x)/(f*cos(e + f*x)^5)
```

3.28 $\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx=-\frac{\sec^3(e+fx)\tan(e+fx)}{f}$$

```
[Out] -sec(f*x+e)^3*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx=-\frac{\tan(e+fx)\sec^3(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]^3*(3 - 4*Sec[e + f*x]^2),x]
```

```
[Out] -((Sec[e + f*x]^3*Tan[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\sec^3(e+fx)\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx=-\frac{\sec^3(e+fx)\tan(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]^3*(3 - 4*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^3*Tan[e + f*x])/f)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result
parallelrisch	$-\frac{8\sin(fx+e)}{f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
risch	$\frac{8i\left(e^{5i(fx+e)}-e^{3i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}$
derivativedivides	$\frac{\frac{3\sec(fx+e)\tan(fx+e)}{2}+4\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)}{f}$
default	$\frac{\frac{3\sec(fx+e)\tan(fx+e)}{2}+4\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)}{f}$
norman	$\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
parts	$\frac{3\sec(fx+e)\tan(fx+e)}{2f}+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{2f}-\frac{4\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx\ldots}{}\right.}{f}$

```
[In] int(sec(f*x+e)^3*(3-4*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] -8/f*sin(f*x+e)/(cos(4*f*x+4*e)+4*cos(2*f*x+2*e)+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx=-\frac{\sin(fx+e)}{f\cos(fx+e)^4}$$

```
[In] integrate(sec(f*x+e)^3*(3-4*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx = -\int \left(-3\sec^3{(e+fx)}\right)\,dx - \int 4\sec^5{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**3*(3-4*sec(f*x+e)**2),x)
[Out] -Integral(-3*sec(e + f*x)**3, x) - Integral(4*sec(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx = -\frac{\sin{(fx+e)}}{\left(\sin{(fx+e)}^4-2\,\sin{(fx+e)}^2+1\right)f}$$

```
[In] integrate(sec(f*x+e)^3*(3-4*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -sin(f*x + e)/((sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx = -\frac{\sin{(fx+e)}}{\left(\sin{(fx+e)}^2-1\right)^2 f}$$

```
[In] integrate(sec(f*x+e)^3*(3-4*sec(f*x+e)^2),x, algorithm="giac")
[Out] -sin(f*x + e)/((sin(f*x + e)^2 - 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sec^3(e+fx)\left(3-4\sec^2(e+fx)\right)\,dx = -\frac{\sin{(e+f\,x)}}{f\,\left(\sin{(e+f\,x)}^2-1\right)^2}$$

```
[In] int(-(4/cos(e + f*x)^2 - 3)/cos(e + f*x)^3,x)
[Out] -sin(e + f*x)/(f*(sin(e + f*x)^2 - 1)^2)
```

3.29 $\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx=-\frac{\sec^2(e+fx)\tan(e+fx)}{f}$$

```
[Out] -sec(f*x+e)^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx=-\frac{\tan(e+fx)\sec^2(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]^2*(2 - 3*Sec[e + f*x]^2),x]
```

```
[Out] -((Sec[e + f*x]^2*Tan[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\sec^2(e+fx)\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx=-\frac{\sec^2(e+fx)\tan(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]^2*(2 - 3*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]^2*Tan[e + f*x])/f)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$-\frac{4\sin(fx+e)}{f(\cos(3fx+3e)+3\cos(fx+e))}$	32
derivativedivides	$\frac{2\tan(fx+e)+3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	34
default	$\frac{2\tan(fx+e)+3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	34
parts	$\frac{2\tan(fx+e)}{f}+\frac{3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	36
risch	$\frac{4i\left(e^{4i(fx+e)}-e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^3}$	41
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$	64

```
[In] int(sec(f*x+e)^2*(2-3*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] -4/f*sin(f*x+e)/(cos(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx=-\frac{\sin(fx+e)}{f\cos(fx+e)^3}$$

```
[In] integrate(sec(f*x+e)^2*(2-3*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx = -\int \left(-2\sec^2(e+fx)\right)\,dx - \int 3\sec^4(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)**2*(2-3*sec(f*x+e)**2),x)
[Out] -Integral(-2*sec(e + f*x)**2, x) - Integral(3*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx = -\frac{\tan(fx+e)^3+\tan(fx+e)}{f}$$

```
[In] integrate(sec(f*x+e)^2*(2-3*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(tan(f*x + e)^3 + tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx = -\frac{\tan(fx+e)^3+\tan(fx+e)}{f}$$

```
[In] integrate(sec(f*x+e)^2*(2-3*sec(f*x+e)^2),x, algorithm="giac")
[Out] -(tan(f*x + e)^3 + tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sec^2(e+fx)\left(2-3\sec^2(e+fx)\right)\,dx = -\frac{\tan(e+f\,x)\,\left(\tan(e+f\,x)^2+1\right)}{f}$$

```
[In] int(-(3/cos(e + f*x)^2 - 2)/cos(e + f*x)^2,x)
[Out] -(tan(e + f*x)*(tan(e + f*x)^2 + 1))/f
```

3.30 $\int \sec(e+fx) \left(1-2\sec^2(e+fx)\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \sec(e+fx) \left(1-2\sec^2(e+fx)\right) \, dx = -\frac{\sec(e+fx)\tan(e+fx)}{f}$$

```
[Out] -sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4128}

$$\int \sec(e+fx) \left(1-2\sec^2(e+fx)\right) \, dx = -\frac{\tan(e+fx)\sec(e+fx)}{f}$$

```
[In] Int[Sec[e + f*x]*(1 - 2*Sec[e + f*x]^2),x]
```

```
[Out] -((Sec[e + f*x]*Tan[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral} = -\frac{\sec(e+fx)\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = -\frac{\sec(e+fx)\tan(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]*(1 - 2*Sec[e + f*x]^2),x]
[Out] -((Sec[e + f*x]*Tan[e + f*x])/f)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\sec(fx+e)\tan(fx+e)}{f}$	18
default	$-\frac{\sec(fx+e)\tan(fx+e)}{f}$	18
parts	$-\frac{\sec(fx+e)\tan(fx+e)}{f}$	18
parallelrisch	$-\frac{2\sin(fx+e)}{f(1+\cos(2fx+2e))}$	25
risch	$\frac{2i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}$	41
norman	$\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}$	48

```
[In] int(sec(f*x+e)*(1-2*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] -sec(f*x+e)*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = -\frac{\sin(fx+e)}{f\cos(fx+e)^2}$$

```
[In] integrate(sec(f*x+e)*(1-2*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = -\int (-\sec(e+fx))\,dx - \int 2\sec^3(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(1-2*sec(f*x+e)**2),x)
[Out] -Integral(-sec(e + f*x), x) - Integral(2*sec(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = \frac{\sin(fx+e)}{\left(\sin(fx+e)^2-1\right)f}$$

```
[In] integrate(sec(f*x+e)*(1-2*sec(f*x+e)^2),x, algorithm="maxima")
[Out] sin(f*x + e)/((sin(f*x + e)^2 - 1)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = -\frac{1}{f\left(\frac{1}{\sin(fx+e)}-\sin(fx+e)\right)}$$

```
[In] integrate(sec(f*x+e)*(1-2*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/(f*(1/sin(f*x + e) - sin(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sec(e+fx)\left(1-2\sec^2(e+fx)\right)\,dx = \frac{\sin(e+f\,x)}{f\,\left(\sin(e+f\,x)^2-1\right)}$$

```
[In] int(-(2/cos(e + f*x)^2 - 1)/cos(e + f*x),x)
[Out] sin(e + f*x)/(f*(sin(e + f*x)^2 - 1))
```

3.31 $\int -\sec^2(e+fx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(e+fx)}{f}$$

```
[Out] -tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3852, 8}

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(e+fx)}{f}$$

```
[In] Int[-Sec[e + f*x]^2,x]
```

```
[Out] -(Tan[e + f*x]/f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{f}$$
$$= -\frac{\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(e+fx)}{f}$$

```
[In] Integrate[-Sec[e + f*x]^2,x]
[Out] -(Tan[e + f*x]/f)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\tan(fx+e)}{f}$	12
default	$-\frac{\tan(fx+e)}{f}$	12
risch	$-\frac{2i}{f\left(e^{2i(fx+e)}+1\right)}$	20
norman	$\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	30
parallelrisch	$\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	30

```
[In] int(-sec(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] -tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int -\sec^2(e+fx)\,dx = -\frac{\sin(fx+e)}{f\cos(fx+e)}$$

```
[In] integrate(-sec(f*x+e)^2,x, algorithm="fricas")
[Out] -sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F]

$$\int -\sec^2(e+fx)\,dx = -\int \sec^2(e+fx)\,dx$$

```
[In] integrate(-sec(f*x+e)**2,x)
[Out] -Integral(sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(fx+e)}{f}$$

```
[In] integrate(-sec(f*x+e)^2,x, algorithm="maxima")
[Out] -tan(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(fx+e)}{f}$$

```
[In] integrate(-sec(f*x+e)^2,x, algorithm="giac")
[Out] -tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\sec^2(e+fx)\,dx = -\frac{\tan(e+f\,x)}{f}$$

```
[In] int(-1/cos(e + f*x)^2,x)
[Out] -tan(e + f*x)/f
```

3.32 $\int -\cos(e+fx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int -\cos(e+fx)\,dx = -\frac{\sin(e+fx)}{f}$$

```
[Out] -sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2717\}$

$$\int -\cos(e+fx)\,dx = -\frac{\sin(e+fx)}{f}$$

```
[In] Int[-Cos[e + f*x],x]
```

```
[Out] -(Sin[e + f*x]/f)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\sin(e+fx)}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int -\cos(e+fx)\,dx = -\frac{\cos(fx)\sin(e)}{f} - \frac{\cos(e)\sin(fx)}{f}$$

```
[In] Integrate[-Cos[e + f*x],x]
[Out] -((Cos[f*x]*Sin[e])/f) - (Cos[e]*Sin[f*x])/f
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\sin(fx+e)}{f}$	12
default	$-\frac{\sin(fx+e)}{f}$	12
risch	$-\frac{\sin(fx+e)}{f}$	12
parallelrisch	$-\frac{\sin(fx+e)}{f}$	12
parts	$-\frac{\sin(fx+e)}{f}$	12
norman	$-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}$	30
meijerg	$-\frac{\cos(e)\sin(fx)}{f}+\frac{\sin(e)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(fx)}{\sqrt{\pi}}\right)}{f}$	35

```
[In] int(-cos(f*x+e),x,method=_RETURNVERBOSE)
[Out] -sin(f*x+e)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\cos(e+fx)\,dx = -\frac{\sin(fx+e)}{f}$$

```
[In] integrate(-cos(f*x+e),x, algorithm="fricas")
[Out] -sin(f*x + e)/f
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int -\cos(e+fx)\,dx = -\begin{cases} \frac{\sin(e+fx)}{f} & \text{for } f \neq 0 \\ x\cos(e) & \text{otherwise} \end{cases}$$

```
[In] integrate(-cos(f*x+e),x)
[Out] -Piecewise((sin(e + f*x)/f, Ne(f, 0)), (x*cos(e), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\cos(e+fx)\,dx = -\frac{\sin(fx+e)}{f}$$

```
[In] integrate(-cos(f*x+e),x, algorithm="maxima")
[Out] -sin(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\cos(e+fx)\,dx = -\frac{\sin(fx+e)}{f}$$

```
[In] integrate(-cos(f*x+e),x, algorithm="giac")
[Out] -sin(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int -\cos(e+fx)\,dx = -\frac{\sin(e+f\,x)}{f}$$

```
[In] int(-cos(e + f*x),x)
```

```
[Out] -sin(e + f*x)/f
```

3.33 $\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx=-\frac{\cos(e+fx)\sin(e+fx)}{f}$$

```
[Out] -cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {4128}

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)\cos(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^2*(-2 + Sec[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]*Sin[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\cos(e+fx)\sin(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx=-\frac{\cos(2fx)\sin(2e)}{2f}-\frac{\cos(2e)\sin(2fx)}{2f}$$

```
[In] Integrate[Cos[e + f*x]^2*(-2 + Sec[e + f*x]^2),x]
[Out] -1/2*(Cos[2*f*x]*Sin[2*e])/f - (Cos[2*e]*Sin[2*f*x])/(2*f)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\sin(2fx+2e)}{2f}$	15
parallelrisch	$-\frac{\sin(2fx+2e)}{2f}$	15
derivativedivides	$-\frac{\cos(fx+e)\sin(fx+e)}{f}$	18
default	$-\frac{\cos(fx+e)\sin(fx+e)}{f}$	18
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	79

```
[In] int(cos(f*x+e)^2*(-2+sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] -1/2/f*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx=-\frac{\cos\left(fx+e\right)\sin\left(fx+e\right)}{f}$$

```
[In] integrate(cos(f*x+e)^2*(-2+sec(f*x+e)^2),x, algorithm="fricas")
[Out] -cos(f*x + e)*sin(f*x + e)/f
```

Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx = x-2\left(\begin{cases}\frac{x\sin^2{(e+fx)}}{2}+\frac{x\cos^2{(e+fx)}}{2}+\frac{\sin{(e+fx)}\cos{(e+fx)}}{2f} & \text{for } f\neq 0\\ x\cos^2{(e)} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(cos(f*x+e)**2*(-2+sec(f*x+e)**2),x)
[Out] x - 2*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*c
os(e + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx = -\frac{\tan{(fx+e)}}{\left(\tan{(fx+e)}^2+1\right)f}$$

```
[In] integrate(cos(f*x+e)^2*(-2+sec(f*x+e)^2),x, algorithm="maxima")
[Out] -tan(f*x + e)/((tan(f*x + e)^2 + 1)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx = -\frac{\sin{(2\,fx+2\,e)}}{2\,f}$$

```
[In] integrate(cos(f*x+e)^2*(-2+sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos^2(e+fx)\left(-2+\sec^2(e+fx)\right)\,dx=-\frac{\sin\left(2\,e+2\,f\,x\right)}{2\,f}$$

```
[In] int(cos(e + f*x)^2*(1/cos(e + f*x)^2 - 2),x)
[Out] -sin(2*e + 2*f*x)/(2*f)
```

3.34 $\int \cos^3(e+fx)\left(-3+2\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cos^3(e+fx)\left(-3+2\sec^2(e+fx)\right)\,dx=-\frac{\cos^2(e+fx)\sin(e+fx)}{f}$$

```
[Out] -cos(f*x+e)^2*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \cos^3(e+fx)\left(-3+2\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)\cos^2(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^3*(-3 + 2*Sec[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]^2*Sin[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\cos^2(e+fx)\sin(e+fx)}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \cos^3(e+fx)\left(-3+2\sec^2(e+fx)\right)\,dx=\frac{2\cos(fx)\sin(e)}{f}+\frac{2\cos(e)\sin(fx)}{f}-\frac{3\sin(e+fx)}{f}+\frac{\sin^3(e+fx)}{f}$$

```
[In] Integrate[Cos[e + f*x]^3*(-3 + 2*Sec[e + f*x]^2),x]
[Out] (2*Cos[f*x]*Sin[e])/f + (2*Cos[e]*Sin[f*x])/f - (3*Sin[e + f*x])/f + Sin[e
+ f*x]^3/f
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

method	result	size
parallelrisch	$\frac{-\sin(fx+e)-\sin(3fx+3e)}{4f}$	26
risch	$-\frac{\sin(fx+e)}{4f}-\frac{\sin(3fx+3e)}{4f}$	27
derivativedivides	$\frac{-\left(2+\cos(fx+e)^2\right)\sin(fx+e)+2\sin(fx+e)}{f}$	32
default	$\frac{-\left(2+\cos(fx+e)^2\right)\sin(fx+e)+2\sin(fx+e)}{f}$	32
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	95

```
[In] int(cos(f*x+e)^3*(-3+2*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/4*(-sin(f*x+e)-sin(3*f*x+3*e))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^3(e+fx) \left(-3+2\sec^2(e+fx)\right) \, dx = -\frac{\cos{(fx+e)}^2\sin{(fx+e)}}{f}$$

```
[In] integrate(cos(f*x+e)^3*(-3+2*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -cos(f*x + e)^2*sin(f*x + e)/f
```

Sympy [F]

$$\int \cos^3(e+fx) \left(-3+2\sec^2(e+fx)\right) \, dx = \int \left(2\sec^2{(e+fx)}-3\right)\cos^3{(e+fx)}\, dx$$

```
[In] integrate(cos(f*x+e)**3*(-3+2*sec(f*x+e)**2),x)
[Out] Integral((2*sec(e + f*x)**2 - 3)*cos(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^3(e+fx) \left(-3+2\sec^2(e+fx)\right) \, dx = \frac{\sin{(fx+e)}^3-\sin{(fx+e)}}{f}$$

```
[In] integrate(cos(f*x+e)^3*(-3+2*sec(f*x+e)^2),x, algorithm="maxima")
[Out] (sin(f*x + e)^3 - sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^3(e+fx) \left(-3+2\sec^2(e+fx)\right) \, dx = \frac{\sin{(fx+e)}^3-\sin{(fx+e)}}{f}$$

```
[In] integrate(cos(f*x+e)^3*(-3+2*sec(f*x+e)^2),x, algorithm="giac")
[Out] (sin(f*x + e)^3 - sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \cos^3(e+fx)\left(-3+2\sec^2(e+fx)\right)\,dx=-\frac{\sin\left(e+f\,x\right)-\sin\left(e+f\,x\right)^3}{f}$$

```
[In] int(cos(e + f*x)^3*(2/cos(e + f*x)^2 - 3),x)
[Out] -(sin(e + f*x) - sin(e + f*x)^3)/f
```

3.35 $\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx=-\frac{\cos^3(e+fx)\sin(e+fx)}{f}$$

```
[Out] -cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)\cos^3(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^4*(-4 + 3*Sec[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]^3*Sin[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\cos^3(e+fx)\sin(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \cos^4(e+fx) \left(-4+3\sec^2(e+fx)\right) \, dx = -\frac{\sin(2(e+fx))}{4f} - \frac{\sin(4(e+fx))}{8f}$$

```
[In] Integrate[Cos[e + f*x]^4*(-4 + 3*Sec[e + f*x]^2),x]
[Out] -1/4*Sin[2*(e + f*x)]/f - Sin[4*(e + f*x)]/(8*f)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

method	result	size
parallelrisch	$\frac{-\sin(4fx+4e)-2\sin(2fx+2e)}{8f}$	29
risch	$-\frac{\sin(4fx+4e)}{8f}-\frac{\sin(2fx+2e)}{4f}$	30
derivativedivides	$\frac{-\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)+\frac{3\cos(fx+e)\sin(fx+e)}{2}}{f}$	45
default	$\frac{-\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)+\frac{3\cos(fx+e)\sin(fx+e)}{2}}{f}$	45
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+\frac{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	111

```
[In] int(cos(f*x+e)^4*(-4+3*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/8*(-sin(4*f*x+4*e)-2*sin(2*f*x+2*e))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^4(e+fx) \left(-4+3\sec^2(e+fx)\right) \, dx = -\frac{\cos\left(fx+e\right)^3\sin\left(fx+e\right)}{f}$$

```
[In] integrate(cos(f*x+e)^4*(-4+3*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -cos(f*x + e)^3*sin(f*x + e)/f
```

Sympy [F]

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx = \int \left(3\sec^2\left(e+fx\right)-4\right)\cos^4\left(e+fx\right)dx$$

```
[In] integrate(cos(f*x+e)**4*(-4+3*sec(f*x+e)**2),x)
[Out] Integral((3*sec(e + f*x)**2 - 4)*cos(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx = -\frac{\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1\right)f}$$

```
[In] integrate(cos(f*x+e)^4*(-4+3*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -tan(f*x + e)/((tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx = -\frac{\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)^2 f}$$

```
[In] integrate(cos(f*x+e)^4*(-4+3*sec(f*x+e)^2),x, algorithm="giac")
[Out] -tan(f*x + e)/((tan(f*x + e)^2 + 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^4(e+fx)\left(-4+3\sec^2(e+fx)\right)\,dx = -\frac{\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)}{f}$$

```
[In] int(cos(e + f*x)^4*(3/cos(e + f*x)^2 - 4),x)
[Out] -(cos(e + f*x)^3*sin(e + f*x))/f
```

3.36 $\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\cos^4(e+fx)\sin(e+fx)}{f}$$

```
[Out] -cos(f*x+e)^4*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4128}

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)\cos^4(e+fx)}{f}$$

```
[In] Int[Cos[e + f*x]^5*(-5 + 4*Sec[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]^4*Sin[e + f*x])/f)
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\text{integral}=-\frac{\cos^4(e+fx)\sin(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\sin(e+fx)}{f}+\frac{2\sin^3(e+fx)}{f}-\frac{\sin^5(e+fx)}{f}$$

```
[In] Integrate[Cos[e + f*x]^5*(-5 + 4*Sec[e + f*x]^2),x]
[Out] -(Sin[e + f*x]/f) + (2*Sin[e + f*x]^3)/f - Sin[e + f*x]^5/f
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

method	result	size
parallelrisch	$\frac{-2\sin(fx+e)-\sin(5fx+5e)-3\sin(3fx+3e)}{16f}$	37
risch	$-\frac{\sin(fx+e)}{8f}-\frac{\sin(5fx+5e)}{16f}-\frac{3\sin(3fx+3e)}{16f}$	41
derivativedivides	$\frac{-\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)+\frac{4\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}}{f}$	52
default	$\frac{-\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)+\frac{4\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}}{f}$	52
norman	$\frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+\frac{20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	127

```
[In] int(cos(f*x+e)^5*(-5+4*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/16*(-2*sin(f*x+e)-sin(5*f*x+5*e)-3*sin(3*f*x+3*e))/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\cos(fx+e)^4\sin(fx+e)}{f}$$

```
[In] integrate(cos(f*x+e)^5*(-5+4*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -cos(f*x + e)^4*sin(f*x + e)/f
```

Sympy [F]

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=\int \left(4\sec^2{(e+fx)}-5\right)\cos^5{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**5*(-5+4*sec(f*x+e)**2),x)
[Out] Integral((4*sec(e + f*x)**2 - 5)*cos(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\sin{(fx+e)}^5-2\,\sin{(fx+e)}^3+\sin{(fx+e)}}{f}$$

```
[In] integrate(cos(f*x+e)^5*(-5+4*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -(sin(f*x + e)^5 - 2*sin(f*x + e)^3 + sin(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\sin{(fx+e)}^5-2\,\sin{(fx+e)}^3+\sin{(fx+e)}}{f}$$

```
[In] integrate(cos(f*x+e)^5*(-5+4*sec(f*x+e)^2),x, algorithm="giac")
[Out] -(sin(f*x + e)^5 - 2*sin(f*x + e)^3 + sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cos^5(e+fx)\left(-5+4\sec^2(e+fx)\right)\,dx=-\frac{\sin{(e+f\,x)}\,\left(\sin{(e+f\,x)}^2-1\right)^2}{f}$$

```
[In] int(cos(e + f*x)^5*(4/cos(e + f*x)^2 - 5),x)
[Out] -(sin(e + f*x)*(sin(e + f*x)^2 - 1)^2)/f
```

3.37 $\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\begin{aligned}&\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\&=\frac{3C\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)}{d}+\frac{3C\sec(c+dx)\tan(c+dx)}{8d}\\&\quad+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}\end{aligned}$$

```
[Out] 3/8*C*arctanh(sin(d*x+c))/d+B*tan(d*x+c)/d+3/8*C*sec(d*x+c)*tan(d*x+c)/d+1/
4*C*sec(d*x+c)^3*tan(d*x+c)/d+1/3*B*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4132, 3852, 12, 3853, 3855}

$$\begin{aligned}&\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\&=\frac{3C\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}\\&\quad+\frac{C\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3C\tan(c+dx)\sec(c+dx)}{8d}\end{aligned}$$

```
[In] Int[Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*ArcTanh[Sin[c + d*x]])/(8*d) + (B*Tan[c + d*x])/d + (3*C*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (B*Tan[c + d*
x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \sec^4(c+dx)\,dx+\int C\sec^5(c+dx)\,dx\\
&= C\int \sec^5(c+dx)\,dx-\frac{B\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{B\tan(c+dx)}{d}+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}+\frac{1}{4}(3C)\int \sec^3(c+dx)\,dx\\
&= \frac{B\tan(c+dx)}{d}+\frac{3C\sec(c+dx)\tan(c+dx)}{8d}+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}\\
&\quad+\frac{B\tan^3(c+dx)}{3d}+\frac{1}{8}(3C)\int \sec(c+dx)\,dx\\
&= \frac{3C\text{arctanh}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)}{d}+\frac{3C\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{9C\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(9C\sec(c+dx)+6C\sec^3(c+dx)+8B(3+\tan^2(c+dx))\right)}{24d}$$

```
[In] Integrate[Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (9*C*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*C*Sec[c + d*x] + 6*C*Sec[c + d
*x]^3 + 8*B*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$-\frac{B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{i\left(9C\,e^{7i(dx+c)}+33C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33C\,e^{3i(dx+c)}-64B\,e^{2i(dx+c)}-9C\,e^{i(dx+c)}-16B\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+\frac{3\ln(e^{i(dx+c)}+$
norman	$\frac{-\frac{(8B-5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(8B+5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(40B-9C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{(40B+9C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}-\frac{3C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8d}$
parallelrisch	$\frac{-18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{12d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \sec^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{9\,C\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,C\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(16\,B\cos\left(dx+c\right)^3+\ldots\right.}{48\,d\cos\left(dx+c\right)^4}$$

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(9*C*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*C*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*B*cos(d*x + c)^3 + 9*C*cos(d*x + c)^2 + 8*B*cos(d*x
 + c) + 6*C)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \int \left(B+C\sec\left(c+dx\right)\right)\sec^4\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sec^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{16\,\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)B-3\,C\left(\frac{2\,\left(3\,\sin\left(dx+c\right)^3-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\ldots\right.\right.}{48\,d}$$

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B - 3*C*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{9\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-4\right.}{}}{24\,d}$$

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(9*C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*C*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(24*B*tan(1/2*d*x + 1/2*c)^7 - 15*C*tan(1/2*d*x + 1/2*c)^7 -
 40*B*tan(1/2*d*x + 1/2*c)^5 - 9*C*tan(1/2*d*x + 1/2*c)^5 + 40*B*tan(1/2*d*
x + 1/2*c)^3 - 9*C*tan(1/2*d*x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1/2*c) - 15*
C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{3\,C\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\left(2\,B-\frac{5\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-\frac{10\,B}{3}-\frac{3\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{10\,B}{3}-\frac{3\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,B-\frac{5\,C}{4}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x)^3,x)
```

```
[Out] (3*C*atanh(tan(c/2 + (d*x)/2)))/(4*d) - (tan(c/2 + (d*x)/2)^7*(2*B - (5*C)/
4) + tan(c/2 + (d*x)/2)^3*((10*B)/3 - (3*C)/4) - tan(c/2 + (d*x)/2)^5*((10*
B)/3 + (3*C)/4) - tan(c/2 + (d*x)/2)*(2*B + (5*C)/4))/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2
)^8 + 1))
```

3.38 $\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{B\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)}{d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{C\tan^3(c+dx)}{3d}$$

```
[Out] 1/2*B*arctanh(sin(d*x+c))/d+C*tan(d*x+c)/d+1/2*B*sec(d*x+c)*tan(d*x+c)/d+1/
3*C*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4132, 3853, 3855, 12, 3852}

$$\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{B\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}+\frac{C\tan^3(c+dx)}{3d}+\frac{C\tan(c+dx)}{d}$$

```
[In] Int[Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (C*Tan[c + d*x])/d + (B*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (C*Tan[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \sec^3(c+dx)\,dx+\int C\sec^4(c+dx)\,dx\\
&= \frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}B\int \sec(c+dx)\,dx+C\int \sec^4(c+dx)\,dx\\
&= \frac{B\text{arctanh}(\sin(c+dx))}{2d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}\\
&\quad -\frac{C\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{B\text{arctanh}(\sin(c+dx))}{2d}+\frac{C\tan(c+dx)}{d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{C\tan^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{B\text{arctanh}(\sin(c+dx))}{2d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{C\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (C*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{i\left(3B\,e^{5i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4C\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{B\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{B\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{(B-2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{(B+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{-9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+6B\sin(2dx+2c}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-C*(-2/3-1/
3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$$
$$= \frac{3\,B\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,B\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,C\cos\left(dx+c\right)^2 -\right.}{12\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/12*(3*B*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*B*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*C)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \int \left(B+C\sec\left(c+dx\right)\right)\sec^3\left(c+dx\right) dx$$

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sec^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)C-3\,B\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C - 3*B*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(57) = 114$.

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int \sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,C\tan\cdots\right)}{\left(\tan\left(\frac{1}{2}\,d\cdots\right)\right)}}{6\,d}$$

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/6*(3*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*B*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*B*tan(1/2*d*x + 1/2*c)^5 - 6*C*tan(1/2*d*x + 1/2*c)^5 + 4*
C*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c) - 6*C*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{(B-2\,C)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{4\,C\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+(-B-2\,C)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x)^2,x)
```

```
[Out] (B*atanh(tan(c/2 + (d*x)/2)))/d + ((4*C*tan(c/2 + (d*x)/2)^3)/3 - tan(c/2 +
 (d*x)/2)*(B + 2*C) + tan(c/2 + (d*x)/2)^5*(B - 2*C))/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.39 $\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{C\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*C*arctanh(sin(d*x+c))/d+B*tan(d*x+c)/d+1/2*C*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4132, 3852, 8, 12, 3853, 3855}

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{C\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (C*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \sec^2(c+dx)\,dx + \int C\sec^3(c+dx)\,dx \\
&= C\int \sec^3(c+dx)\,dx - \frac{B\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{B\tan(c+dx)}{d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}C\int \sec(c+dx)\,dx \\
&= \frac{C\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{B\tan(c+dx)}{d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{C\text{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (C*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (C*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{B\tan(dx+c)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{B\tan(dx+c)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{B\tan(dx+c)}{d}+\frac{C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{-C(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+C(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2B\sin(2dx+2c)+2C\sin(dx+c)}{2d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{(2B+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2B-C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}-\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{i\left(C\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-C\,e^{i(dx+c)}-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)C}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)C}{2d}$

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(B*tan(d*x+c)+C*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{C\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-C\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,B\cos\left(dx+c\right)+C\right)\mathrm{s}}{4\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/4*(C*cos(d*x + c)^2*log(sin(d*x + c) + 1) - C*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*B*cos(d*x + c) + C)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int\left(B+C\sec\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=-\frac{C\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,B\tan\left(dx+c\right)}{4\,d}$$

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] -1/4*(C*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*B*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(2*B*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(
1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

Mupad [B] (verification not implemented)

Time = 15.82 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{C\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(2\,B-C)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,B+C)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x),x)
```

```
[Out] (C*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(2*B - C) - tan(c/2
 + (d*x)/2)*(2*B + C))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 +
1))
```

3.40 $\int (B\sec(c+dx)+C\sec^2(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (B\sec(c+dx)+C\sec^2(c+dx))\,dx=\frac{B\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{C\tan(c+dx)}{d}$$

```
[Out] B*arctanh(sin(d*x+c))/d+C*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3855, 3852, 8}

$$\int (B\sec(c+dx)+C\sec^2(c+dx))\,dx=\frac{B\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{C\tan(c+dx)}{d}$$

```
[In] Int[B*Sec[c + d*x] + C*Sec[c + d*x]^2,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d + (C*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= B\int \sec(c+dx)\,dx + C\int \sec^2(c+dx)\,dx\\ &= \frac{B\text{arctanh}(\sin(c+dx))}{d} - \frac{C\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{B\text{arctanh}(\sin(c+dx))}{d} + \frac{C\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\text{arctanh}(\sin(c+dx))}{d} + \frac{C\tan(c+dx)}{d}$$

```
[In] Integrate[B*Sec[c + d*x] + C*Sec[c + d*x]^2,x]
[Out] (B*ArcTanh[Sin[c + d*x]])/d + (C*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))+C\tan(dx+c)}{d}$	30
default	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{C\tan(dx+c)}{d}$	32
parts	$\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{C\tan(dx+c)}{d}$	32
risch	$-\frac{B\ln(\mathrm{e}^{i(dx+c)}-i)}{d}+\frac{B\ln(\mathrm{e}^{i(dx+c)}+i)}{d}+\frac{2iC}{d(\mathrm{e}^{2i(dx+c)}+1)}$	59
parallelrisch	$\frac{-B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+C\sin(dx+c)}{d\cos(dx+c)}$	63
norman	$-\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

```
[In] int(B*sec(d*x+c)+C*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(B*ln(sec(d*x+c)+tan(d*x+c))+C*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-B\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,C\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

```
[In] integrate(B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="fricas")
[Out] 1/2*(B*cos(d*x + c)*log(sin(d*x + c) + 1) - B*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*C*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(B+C\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

```
[In] integrate(B*sec(d*x+c)+C*sec(d*x+c)**2,x)
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{C\tan\left(dx+c\right)}{d}$$

```
[In] integrate(B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="maxima")
[Out] B*log(sec(d*x + c) + tan(d*x + c))/d + C*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\left(\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)-2\right|\right)\right)}{4\,d} + \frac{C\tan\left(dx+c\right)}{d}$$

```
[In] integrate(B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*B*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*x + c)
 + sin(d*x + c) - 2)))/d + C*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,C\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

```
[In] int(B/cos(c + d*x) + C/cos(c + d*x)^2,x)
```

```
[Out] (2*B*atanh(tan(c/2 + (d*x)/2)))/d - (2*C*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

3.41 $\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 16

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=Bx+\frac{C\text{arctanh}(\sin(c+dx))}{d}$$

```
[Out] B*x+C*arctanh(sin(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4132, 8, 12, 3855}

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{C\text{arctanh}(\sin(c+dx))}{d}+Bx$$

```
[In] Int[Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] B*x + (C*ArcTanh[Sin[c + d*x]])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= B\int 1\,dx + \int C\sec(c+dx)\,dx \\ &= Bx + C\int \sec(c+dx)\,dx \\ &= Bx + \frac{C\operatorname{arctanh}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = Bx + \frac{C\operatorname{arctanh}(\sin(c+dx))}{d}$$

```
[In] Integrate[Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] B*x + (C*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$\frac{C\ln(\sec(dx+c)+\tan(dx+c))+B(dx+c)}{d}$	29
default	$\frac{C\ln(\sec(dx+c)+\tan(dx+c))+B(dx+c)}{d}$	29
parallelrisch	$\frac{Bxd-C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	39
risch	$Bx+\frac{\ln(e^{i(dx+c)}+i)C}{d}-\frac{\ln(e^{i(dx+c)}-i)C}{d}$	42
norman	$\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-Bx}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}+\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	87

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(C*ln(sec(d*x+c)+tan(d*x+c))+B*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,Bdx + C\log\left(\sin\left(dx+c\right)+1\right) - C\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(2*B*d*x + C*log(sin(d*x + c) + 1) - C*log(-sin(d*x + c) + 1))/d
```

Sympy [F]

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(B + C\sec\left(c+dx\right)\right)\cos\left(c+dx\right)\sec\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,(dx+c)B+C(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(2*(d*x + c)*B + C*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(dx+c)B+C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*B + C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,C\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

```
[In] int(cos(c + d*x)*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*C*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.42 $\int \cos^2(c+dx) (B\sec(c+dx)+C\sec^2(c+dx)) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \cos^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = Cx+\frac{B\sin(c+dx)}{d}$$

```
[Out] C*x+B*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4132, 2717, 8}

$$\int \cos^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{B\sin(c+dx)}{d}+Cx$$

```
[In] Int[Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] C*x + (B*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= B\int \cos(c+dx)\,dx + \int C\,dx \\ &= Cx + \frac{B\sin(c+dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= Cx + \frac{B\cos(dx)\sin(c)}{d} + \frac{B\cos(c)\sin(dx)}{d}$$

```
[In] Integrate[Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] C*x + (B*Cos[d*x]*Sin[c])/d + (B*Cos[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$Cx+\frac{B\sin(dx+c)}{d}$	16
parallelrisch	$\frac{dxC+B\sin(dx+c)}{d}$	18
derivativedivides	$\frac{B\sin(dx+c)+C(dx+c)}{d}$	21
default	$\frac{B\sin(dx+c)+C(dx+c)}{d}$	21
norman	$\frac{Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-Cx-\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	112

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] C*x+B*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{Cdx+B\sin\left(dx+c\right)}{d}$$

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] (C*d*x + B*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int\left(B+C\sec\left(c+dx\right)\right)\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{(dx+c)C+B\sin\left(dx+c\right)}{d}$$

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] ((d*x + c)*C + B*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15)=30$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{(dx+c)C+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*C + 2*B*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^2(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{B\,\sin\left(c+d\,x\right)+C\,d\,x}{d}$$

```
[In] int(cos(c + d*x)^2*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (B*sin(c + d*x) + C*d*x)/d
```

3.43 $\int \cos^3(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \cos^3(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx = \frac{Bx}{2} + \frac{C \sin(c+dx)}{d} + \frac{B \cos(c+dx) \sin(c+dx)}{2d}$$

```
[Out] 1/2*B*x+C*sin(d*x+c)/d+1/2*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4132, 2715, 8, 12, 2717}

$$\int \cos^3(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx = \frac{B \sin(c+dx) \cos(c+dx)}{2d} + \frac{Bx}{2} + \frac{C \sin(c+dx)}{d}$$

```
[In] Int[Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (B*x)/2 + (C*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^2(c+dx)\,dx + \int C\cos(c+dx)\,dx \\
&= \frac{B\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}B\int 1\,dx + C\int \cos(c+dx)\,dx \\
&= \frac{Bx}{2} + \frac{C\sin(c+dx)}{d} + \frac{B\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{4C\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx)))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (4*C*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result
risch	$\frac{Bx}{2}+\frac{C\sin(dx+c)}{d}+\frac{B\sin(2dx+2c)}{4d}$
parallelrisch	$\frac{2Bxd+B\sin(2dx+2c)+4C\sin(dx+c)}{4d}$
derivativedivides	$\frac{B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C\sin(dx+c)}{d}$
default	$\frac{B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C\sin(dx+c)}{d}$
norman	$\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{(B-2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}+\frac{(B+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{Bx}{2}-Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{2}-\frac{(B-2C)\tan\left(\frac{d}{}\right.}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*B*x+C*sin(d*x+c)/d+1/4*B/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{Bdx+(B\cos(dx+c)+2\,C)\sin(dx+c)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(B*d*x + (B*cos(d*x + c) + 2*C)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int (B+C\sec(c+dx))\cos^3(c+dx)\sec(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B+4\,C\sin(dx+c)}{4\,d}$$

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B + 4*C*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \cos^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(dx+c)B-\frac{2\left(B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*((d*x + c)*B - 2*(B*tan(1/2*d*x + 1/2*c)^3 - 2*C*tan(1/2*d*x + 1/2*c)^3
 - B*tan(1/2*d*x + 1/2*c) - 2*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\,x}{2}+\frac{B\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{C\,\sin(c+d\,x)}{d}$$

```
[In] int(cos(c + d*x)^3*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (B*x)/2 + (B*sin(2*c + 2*d*x))/(4*d) + (C*sin(c + d*x))/d
```

3.44 $\int \cos^4(c+dx) \left(B \sec(c+dx)+C \sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \cos^4(c+dx) \left(B \sec(c+dx)+C \sec^2(c+dx)\right) \, dx = \frac{Cx}{2}+\frac{B \sin(c+dx)}{d}+\frac{C \cos(c+dx) \sin(c+dx)}{2d}-\frac{B \sin^3(c+dx)}{3d}$$

```
[Out] 1/2*C*x+B*sin(d*x+c)/d+1/2*C*cos(d*x+c)*sin(d*x+c)/d-1/3*B*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4132, 2713, 12, 2715, 8}

$$\int \cos^4(c+dx) \left(B \sec(c+dx)+C \sec^2(c+dx)\right) \, dx = -\frac{B \sin^3(c+dx)}{3d}+\frac{B \sin(c+dx)}{d}+\frac{C \sin(c+dx) \cos(c+dx)}{2d}+\frac{Cx}{2}$$

```
[In] Int[Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (B*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^3(c+dx)\,dx+\int C\cos^2(c+dx)\,dx\\
&= C\int \cos^2(c+dx)\,dx-\frac{B\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= \frac{B\sin(c+dx)}{d}+\frac{C\cos(c+dx)\sin(c+dx)}{2d}-\frac{B\sin^3(c+dx)}{3d}+\frac{1}{2}C\int 1\,dx\\
&= \frac{Cx}{2}+\frac{B\sin(c+dx)}{d}+\frac{C\cos(c+dx)\sin(c+dx)}{2d}-\frac{B\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\
&=\frac{C(c+dx)}{2d}+\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}+\frac{C\sin(2(c+dx))}{4d}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (C*(c + d*x))/(2*d) + (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d) + (C*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{6dxC+9B\sin(dx+c)+B\sin(3dx+3c)+3\sin(2dx+2c)C}{12d}$
risch	$\frac{Cx}{2}+\frac{3B\sin(dx+c)}{4d}+\frac{B\sin(3dx+3c)}{12d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{\frac{B\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{B\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{(2B-C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{Cx}{2}-\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{3Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-C}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*(6*d*x*C+9*B*sin(d*x+c)+B*sin(3*d*x+3*c)+3*sin(2*d*x+2*c)*C)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,Cdx+\left(2\,B\cos\left(dx+c\right)^2+3\,C\cos\left(dx+c\right)+4\,B\right)\sin\left(dx+c\right)}{6\,d}$$

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/6*(3*C*d*x + (2*B*cos(d*x + c)^2 + 3*C*cos(d*x + c) + 4*B)*sin(d*x + c))/
d
```

Sympy [F]

$$\int \cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(B+C\sec\left(c+dx\right)\right)\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= -\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)B-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)C}{12\,d}$$

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*C)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(48) = 96.

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \cos^4(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\,(dx+c)C+\frac{2\left(6\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/6*(3*(d*x + c)*C + 2*(6*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*
c)^5 + 4*B*tan(1/2*d*x + 1/2*c)^3 + 6*B*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{C\,x}{2}+\frac{2\,B\,\sin\left(c+d\,x\right)}{3\,d}+\frac{C\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{B\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^4*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (C*x)/2 + (2*B*sin(c + d*x))/(3*d) + (C*cos(c + d*x)*sin(c + d*x))/(2*d) +
(B*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.45 $\int \cos^5(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \cos^5(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx = \frac{3Bx}{8} + \frac{C \sin(c+dx)}{d} + \frac{3B \cos(c+dx) \sin(c+dx)}{8d} + \frac{B \cos^3(c+dx) \sin(c+dx)}{4d} - \frac{C \sin^3(c+dx)}{3d}$$

```
[Out] 3/8*B*x+C*sin(d*x+c)/d+3/8*B*cos(d*x+c)*sin(d*x+c)/d+1/4*B*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*C*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4132, 2715, 8, 12, 2713}

$$\int \cos^5(c+dx) \left(B \sec(c+dx) + C \sec^2(c+dx)\right) dx = \frac{B \sin(c+dx) \cos^3(c+dx)}{4d} + \frac{3B \sin(c+dx) \cos(c+dx)}{8d} + \frac{3Bx}{8} - \frac{C \sin^3(c+dx)}{3d} + \frac{C \sin(c+dx)}{d}$$

```
[In] Int[Cos[c + d*x]^5*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*B*x)/8 + (C*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (C*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^4(c+dx)\,dx+\int C\cos^3(c+dx)\,dx\\
&= \frac{B\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3B)\int \cos^2(c+dx)\,dx+C\int \cos^3(c+dx)\,dx\\
&= \frac{3B\cos(c+dx)\sin(c+dx)}{8d}+\frac{B\cos^3(c+dx)\sin(c+dx)}{4d}\\
&\quad+\frac{1}{8}(3B)\int 1\,dx-\frac{C\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= \frac{3Bx}{8}+\frac{C\sin(c+dx)}{d}+\frac{3B\cos(c+dx)\sin(c+dx)}{8d}\\
&\quad+\frac{B\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{C\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$
\int \cos^5(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx
$$
$$
=\frac{3B(c+dx)}{8d}+\frac{C\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}+\frac{B\sin(2(c+dx))}{4d}+\frac{B\sin(4(c+dx))}{32d}
$$

```
[In] Integrate[Cos[c + d*x]^5*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*B*(c + d*x))/(8*d) + (C*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d) + (B*
Sin[2*(c + d*x)])/(4*d) + (B*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{36Bxd+3B\sin(4dx+4c)+24B\sin(2dx+2c)+72C\sin(dx+c)+8C\sin(3dx+3c)}{96d}$
derivativedivides	$\frac{B\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{B\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3Bx}{8}+\frac{3C\sin(dx+c)}{4d}+\frac{B\sin(4dx+4c)}{32d}+\frac{\sin(3dx+3c)C}{12d}+\frac{B\sin(2dx+2c)}{4d}$
norman	$\frac{-\frac{3Bx}{8}-\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{15Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8}+\frac{15Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{2}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8}+\frac{(3B-8C)\tan(}{6d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

```
[In] int(cos(d*x+c)^5*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(36*B*x*d+3*B*sin(4*d*x+4*c)+24*B*sin(2*d*x+2*c)+72*C*sin(d*x+c)+8*C*s
in(3*d*x+3*c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{9\,Bdx + \left(6\,B\cos\left(dx+c\right)^3 + 8\,C\cos\left(dx+c\right)^2 + 9\,B\cos\left(dx+c\right) + 16\,C\right)\sin\left(dx+c\right)}{24\,d}$$

```
[In] integrate(cos(d*x+c)^5*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/24*(9*B*d*x + (6*B*cos(d*x + c)^3 + 8*C*cos(d*x + c)^2 + 9*B*cos(d*x + c)
 + 16*C)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(B + C\sec\left(c+dx\right)\right)\cos^5\left(c+dx\right)\sec\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**5*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)**5*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^5(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) + 8\,\sin\left(2\,dx+2\,c\right))B - 32\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)C}{96\,d}$$

```
[In] integrate(cos(d*x+c)^5*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.84

$$\int \cos^5(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{9\,(dx+c)B - \frac{2\left(15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,C\tan\left(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

```
[In] integrate(cos(d*x+c)^5*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*B - 2*(15*B*tan(1/2*d*x + 1/2*c)^7 - 24*C*tan(1/2*d*x + 1
/2*c)^7 - 9*B*tan(1/2*d*x + 1/2*c)^5 - 40*C*tan(1/2*d*x + 1/2*c)^5 + 9*B*ta
n(1/2*d*x + 1/2*c)^3 - 40*C*tan(1/2*d*x + 1/2*c)^3 - 15*B*tan(1/2*d*x + 1/2
*c) - 24*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^5(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\,B\,x}{8} + \frac{2\,C\,\sin\left(c+d\,x\right)}{3\,d} + \frac{3\,B\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d} + \frac{B\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d} + \frac{C\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^5*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (3*B*x)/8 + (2*C*sin(c + d*x))/(3*d) + (3*B*cos(c + d*x)*sin(c + d*x))/(8*d
) + (B*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (C*cos(c + d*x)^2*sin(c + d*x))
/(3*d)
```

3.46 $\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3Cx}{8}+\frac{B\sin(c+dx)}{d}+\frac{3C\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}$$

```
[Out] 3/8*C*x+B*sin(d*x+c)/d+3/8*C*cos(d*x+c)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^3*sin
(d*x+c)/d-2/3*B*sin(d*x+c)^3/d+1/5*B*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4132, 2713, 12, 2715, 8}

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\sin^5(c+dx)}{5d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3C\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Cx}{8}$$

```
[In] Int[Cos[c + d*x]^6*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*x)/8 + (B*Sin[c + d*x])/d + (3*C*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*B*Sin[c + d*x]^3)/(3*d) + (B*Sin[c
 + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^5(c+dx)\,dx+\int C\cos^4(c+dx)\,dx\\
&= C\int \cos^4(c+dx)\,dx-\frac{B\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= \frac{B\sin(c+dx)}{d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2B\sin^3(c+dx)}{3d}\\
&\quad+\frac{B\sin^5(c+dx)}{5d}+\frac{1}{4}(3C)\int \cos^2(c+dx)\,dx\\
&= \frac{B\sin(c+dx)}{d}+\frac{3C\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}\\
&\quad-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}+\frac{1}{8}(3C)\int 1\,dx
\end{aligned}$$

$$= \frac{3Cx}{8} + \frac{B\sin(c+dx)}{d} + \frac{3C\cos(c+dx)\sin(c+dx)}{8d} + \frac{C\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2B\sin^3(c+dx)}{3d} + \frac{B\sin^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3C(c+dx)}{8d} + \frac{B\sin(c+dx)}{d} - \frac{2B\sin^3(c+dx)}{3d} + \frac{B\sin^5(c+dx)}{5d} + \frac{C\sin(2(c+dx))}{4d} + \frac{C\sin(4(c+dx))}{32d}$$

```
[In] Integrate[Cos[c + d*x]^6*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*(c + d*x))/(8*d) + (B*Sin[c + d*x])/d - (2*B*Sin[c + d*x]^3)/(3*d) + (
B*Sin[c + d*x]^5)/(5*d) + (C*Sin[2*(c + d*x)])/(4*d) + (C*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{180dxC+300B\sin(dx+c)+6B\sin(5dx+5c)+50B\sin(3dx+3c)+15\sin(4dx+4c)C+120\sin(2dx+2c)C}{480d}$
derivativedivides	$\frac{\frac{B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3Cx}{8}+\frac{5B\sin(dx+c)}{8d}+\frac{B\sin(5dx+5c)}{80d}+\frac{\sin(4dx+4c)C}{32d}+\frac{5B\sin(3dx+3c)}{48d}+\frac{\sin(2dx+2c)C}{4d}$
norman	$\frac{C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{3Cx}{8}-\frac{15Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}-\frac{27Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8}-\frac{15Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}+\frac{15Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{27Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{8}$

```
[In] int(cos(d*x+c)^6*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480*(180*d*x*C+300*B*sin(d*x+c)+6*B*sin(5*d*x+5*c)+50*B*sin(3*d*x+3*c)+15
*sin(4*d*x+4*c)*C+120*sin(2*d*x+2*c)*C)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{45\,Cdx+\left(24\,B\cos\left(dx+c\right)^4+30\,C\cos\left(dx+c\right)^3+32\,B\cos\left(dx+c\right)^2+45\,C\cos\left(dx+c\right)+64\,B\right)\sin\left(dx\right.}{120\,d}$$

```
[In] integrate(cos(d*x+c)^6*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/120*(45*C*d*x + (24*B*cos(d*x + c)^4 + 30*C*cos(d*x + c)^3 + 32*B*cos(d*x
 + c)^2 + 45*C*cos(d*x + c) + 64*B)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{32\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)B+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,d\right.\right.}{480\,d}$$

```
[In] integrate(cos(d*x+c)^6*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.67

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

$$=\frac{45\,(dx+c)C+\frac{2\left(120\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

```
[In] integrate(cos(d*x+c)^6*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)*C + 2*(120*B*tan(1/2*d*x + 1/2*c)^9 - 75*C*tan(1/2*d*x
+ 1/2*c)^9 + 160*B*tan(1/2*d*x + 1/2*c)^7 - 30*C*tan(1/2*d*x + 1/2*c)^7 + 4
64*B*tan(1/2*d*x + 1/2*c)^5 + 160*B*tan(1/2*d*x + 1/2*c)^3 + 30*C*tan(1/2*d
*x + 1/2*c)^3 + 120*B*tan(1/2*d*x + 1/2*c) + 75*C*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \cos^6(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{3\,C\,x}{8}$$

$$+\frac{\left(2\,B-\frac{5\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,B}{3}-\frac{C}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(\frac{8\,B}{3}+\frac{C}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(2}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

```
[In] int(cos(c + d*x)^6*(B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (3*C*x)/8 + (tan(c/2 + (d*x)/2)^3*((8*B)/3 + C/2) + tan(c/2 + (d*x)/2)^9*(2
*B - (5*C)/4) + tan(c/2 + (d*x)/2)^7*((8*B)/3 - C/2) + (116*B*tan(c/2 + (d*
x)/2)^5)/15 + tan(c/2 + (d*x)/2)*(2*B + (5*C)/4))/(d*(tan(c/2 + (d*x)/2)^2
+ 1)^5)
```

3.47 $\int (b\sec(c+dx))^{3/2} \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 169

$$\begin{aligned}
&\int (b\sec(c+dx))^{3/2} \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \\
&-\frac{6b^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&+\frac{6bC\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+\frac{2C(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd}
\end{aligned}$$

```
[Out] 2/3*B*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*C*(b*sec(d*x+c))^(5/2)*sin(d*x+
c)/b/d-6/5*b^2*C*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+6/5*b*C
*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4132, 3853, 3856, 2720, 12, 16, 2719}

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = -\frac{6b^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2C\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd}+\frac{6bC\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

```
[In] Int[(b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-6*b^2*C*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
 d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c
+ d*x]])/(3*d) + (6*b*C*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*B*(b*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*C*(b*Sec[c + d*x])^(5/2)*Sin[c
 + d*x])/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int (b\sec(c+dx))^{5/2}\,dx}{b} + \int C\sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx \\
&= \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{1}{3}(bB)\int\sqrt{b\sec(c+dx)}\,dx + C\int\sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx \\
&= \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{C\int (b\sec(c+dx))^{7/2}\,dx}{b^2} \\
&\quad + \frac{1}{3}\left(bB\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{2C(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} + \frac{1}{5}(3C)\int (b\sec(c+dx))^{3/2}\,dx \\
&= \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{6bC\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{2C(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{1}{5}\left(3b^2C\right)\int\frac{1}{\sqrt{b\sec(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{6bC\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2C(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{(3b^2C)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$= -\frac{6b^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{6bC\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2C(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(b\sec(c+dx))^{5/2}\left(-36C\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20B\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\right.}{30bd}$$

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] ((b*Sec[c + d*x])^(5/2)*(-36*C*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
 + 20*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*C*Sin[c + d*x] +
10*B*Sin[2*(c + d*x)] + 9*C*Sin[3*(c + d*x)]))/(30*b*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.28

method	result
parts	$\frac{2B\sqrt{b\sec(dx+c)}\,b\left(-i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\cos(dx+c)-i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+}{\cos(dx+c}}\right.}{3d}$
default	$-\frac{2b\sqrt{b\sec(dx+c)}\left(5iB\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)^2-9iC\,\text{EllipticF}(i(-\cot(d\right.}{}$

```
[In] int((b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/3*B/d*(b*sec(d*x+c))^(1/2)*b*(-I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d
*x+c)+csc(d*x+c)),I)+tan(d*x+c))+2/5*C/d*(b*sec(d*x+c))^(1/2)*b/(cos(d*x+c)
+1)*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)^2-3*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos
(d*x+c)^2+6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)-6*I*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*
cos(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-3*I*EllipticF(I*(cot(d*x+c)-csc(d*x+c
)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*sin(d*x+
c)+tan(d*x+c)+sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{-5i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i}{}$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*C*b^(3/2)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + 9*I*sqrt(2)*C*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*C*
b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*C*b)*sqrt(b/cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int (b\sec(c+dx))^{\frac{3}{2}}(B+C\sec(c+dx))\sec(c+dx)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((b*sec(c + d*x))**(3/2)*(B + C*sec(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)(b\sec(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)(b\sec(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{3/2} \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(\frac{B}{\cos(c+d\,x)} + \frac{C}{\cos(c+d\,x)^2}\right)\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2),x)
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2), x)
```

3.48 $\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 135

$$\begin{aligned}\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx &= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &\quad+\frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}\\ &\quad+\frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}+\frac{2C(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd}\end{aligned}$$

```
[Out] 2/3*C*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b/d-2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+2*B*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/3*C*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used

$= \{4132, 3853, 3856, 2719, 12, 16, 2720\}$

$$\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{d} - \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\sin(c+dx)(b\sec(c+dx))^{3/2}}{3bd} + \frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}$$

```
[In] Int[Sqrt[b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*b*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]
])/(3*d) + (2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*C*(b*Sec[c + d*x]
)^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int (b\sec(c+dx))^{3/2}\,dx}{b} + \int C\sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$= \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - (bB)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx + C\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$= \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{C\int (b\sec(c+dx))^{5/2}\,dx}{b^2} - \frac{(bB)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}C\int \sqrt{b\sec(c+dx)}\,dx$$

$$= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}\left(C\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$

$$= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(b\sec(c+dx))^{3/2}\left(-6B\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2C\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2(C+\right.}{3bd}$$

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ 2*C*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(C + 3*B*Cos[c + d*x
])*Sin[c + d*x]))/(3*b*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.96

method	result
parts	$\frac{2B\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\cos(dx+c)^2-i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticE}(i(-\right.}{}$
default	$\frac{2\sqrt{b\sec(dx+c)}\left(3iB\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)^2-3iB\operatorname{EllipticE}(i(-\cot(dx+c)-\right.}{}$

```
[In] int((b*sec(d*x+c))^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*B/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)^2-I*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos
(d*x+c)^2+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I
)*cos(d*x+c)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)+sin(d*x+c)
)*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)-2/3*C/d*(b*sec(d*x+c))^(1/2)*(I*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x
+c)+csc(d*x+c)),I)*cos(d*x+c)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.36

$$\int \sqrt{b\sec(c+dx)}\big(B\sec(c+dx)+C\sec^2(c+dx)\big)\,dx$$
$$= \frac{-i\sqrt{2}C\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}C\sqrt{b}\cos(dx+c)}{}$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*C*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*C*sqrt(b)*cos(d*x + c)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c)
+ C)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{b\sec(c+dx)}\big(B\sec(c+dx)+C\sec^2(c+dx)\big)\,dx$$
$$= \int \sqrt{b\sec(c+dx)}(B+C\sec(c+dx))\sec(c+dx)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*(B + C*sec(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b\sec(c+dx)}\big(B\sec(c+dx)+C\sec^2(c+dx)\big)\,dx$$
$$= \int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)\sqrt{b\sec(dx+c)}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(\frac{B}{\cos\left(c+d\,x\right)}+\frac{C}{{\cos\left(c+d\,x\right)}^2}\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2), x)
```

3.49 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 109

$$
\begin{aligned}
\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx
&= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&\quad+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} \\
&\quad+\frac{2C\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd}
\end{aligned}
$$

```
[Out] -2*C*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*C*sin(d*x+c)*(b*s
ec(d*x+c))^(1/2)/b/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b
/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used

$= \{4132, 3856, 2720, 12, 16, 3853, 2719\}$

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$
$$+ \frac{2C\sin(c+dx)\sqrt{b\sec(c+dx)}}{bd} - \frac{2CE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(b*d) + (2*C*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int\sqrt{b\sec(c+dx)}\,dx}{b} + \int\frac{C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

$$= C\int\frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx + \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{C\int(b\sec(c+dx))^{3/2}\,dx}{b^2}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd} - C\int\frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{C\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\left(-CE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{C\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
[Out] (2*(-(C*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (C*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.22

method	result
parts	$-\frac{2iB\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{d\sqrt{b\sec(dx+c)}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}-\frac{2C\left(i\,\text{EllipticE}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \sqrt{\frac{\cos(\ldots}{\cos(d\ldots}}\right.}{}$
default	$-\frac{2\left(iB\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \cos(dx+c)-iC\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\ldots\right.}{}$

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
[Out] -2*I*B/d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)/(
b*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*C/d/(cos(d*x+c)+1)/
(b*sec(d*x+c))^(1/2)*(I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*EllipticF(I*(-
cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I
)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*
(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)
-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0}{}$$

[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="
fricas")

[Out] (-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - I*sqrt(2)*C*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*C*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*C*s
qrt(b/cos(d*x + c))*sin(d*x + c))/(b*d)

Sympy [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{(B+C\sec(c+dx))\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)

[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/sqrt(b*sec(c + d*x)), x)

Maxima [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{\sqrt{b\sec(dx+c)}}\,dx$$

[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="
maxima")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(b*sec(d*x + c)), x)

Giac [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="
giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

3.50 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 85

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*C*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4132, 3856, 2719, 12, 16, 2720}

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]
)/(b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}+\int \frac{C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx\\
&= C\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx+\frac{B\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{C\int \sqrt{b\sec(c+dx)}\,dx}{b^2}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\left(C\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}
\end{aligned}$$

$$=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int\frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2\left(BE\left(\frac{1}{2}(c+dx)\middle|2\right)+C\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
[Out] (2*(B*EllipticE[(c + d*x)/2, 2] + C*EllipticF[(c + d*x)/2, 2]))/(b*d*Sqrt[C
os[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 414, normalized size of antiderivative = 4.87

method	result
risch	$-\frac{iB\sqrt{2}}{db\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{iC\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}+B\left(-\frac{2\left(b\,e^{2i(dx+\cdots}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2\cdots}}\right.\right.}{\cdots}$
parts	$\frac{2B\left(i\,\text{EllipticE}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)-i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cdots\right.}{\cdots}$
default	$\frac{2iB\sqrt{\left((1-\cos(dx+c))^2\csc(dx+c)^2+1\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\sqrt{-(1-\cos(dx+c))^2\csc\cdots}}{\cdots}$

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
[Out] -I*B/d*2^(1/2)/b/(b*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-I/d*(I*C*(-I
*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d
*x+c)))^(1/2)/(b*exp(I*(d*x+c))^3+b*exp(I*(d*x+c)))^(1/2)*EllipticF((-I*(ex
p(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+B*(-2*(b*exp(I*(d*x+c))^2+b)/b/(exp(I*(
d*x+c))*(b*exp(I*(d*x+c))^2+b))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/
2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(b*exp(I*(d*x+c))^
3+b*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2
*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)))))*2^(1/2)
```

```
/b/(exp(I*(d*x+c))^2+1)/(b*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(b*ex
p(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}C\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})-}{}$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*C*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*C*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d
)
```

Sympy [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{(B+C\sec{(c+dx)})\sec{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\sec{(dx+c)}^2+B\sec{(dx+c)}}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\sec{(dx+c)}^2+B\sec{(dx+c)}}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\frac{B}{\cos(c+d\,x)}+\frac{C}{{\cos(c+d\,x)}^2}}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

3.51 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 116

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2CE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*B*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)+2*C*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)
^(1/2)/(b*sec(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c)
)^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4132, 3854, 3856, 2720, 12, 16, 2719}

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{2CE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x
]])/(3*b^3*d) + (2*B*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b}+\int \frac{C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx\\
&= \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{B\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}+C\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx\\
&= \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{C\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b^2}+\frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3}\\
&= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d}\\
&\quad+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{C\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&\quad+\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$
\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)+2B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{B\sin(2(c+dx))}{\sqrt{\cos(c+dx)}}}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}
$$

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*C*EllipticE[(c + d*x)/2, 2] + 2*B*EllipticF[(c + d*x)/2, 2] + (B*Sin[2*(
c + d*x)])/Sqrt[Cos[c + d*x]])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.62

method	result
parts	$-\frac{2B\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx\right.}{3d\sqrt{b\sec(dx+c)}\,b^2}$
default	$-\frac{2\left(iB\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+3iC\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c))\right.}{}$

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*B/d/(b*sec(d*x+c))^(1/2)/b^2*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc
(d*x+c)),I)*sec(d*x+c)-sin(d*x+c))+2*C/b^2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^
(1/2)*(I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*EllipticF(I*(-cot(d*x+c)+csc(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icE(I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+cs
c(d*x+c)),I)*sec(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)+sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\,B\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-i\sqrt{2}B\sqrt{b}\text{weierstrassPInve}}{}$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*B*sqrt(
b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*
sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqr
t(2)*C*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*I*sqrt(2)*C*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{(B+C\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="
maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="
giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

3.52 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 147

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^4d} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{3b^3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/5*B*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(3/2)+2/3*C*sin(d*x+c)/b^3/d/(b*sec(d
*x+c))^(1/2)+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2
/3*C*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4132, 3854, 3856, 2719, 12, 16, 2720}

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^4d} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\sin(c+dx)}{3b^3d\sqrt{b\sec(c+dx)}} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}$$

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d
*x]])/(3*b^4*d) + (2*B*Sin[c + d*x])/(5*b^2*d*(b*Sec[c + d*x])^(3/2)) + (2*
C*Sin[c + d*x])/(3*b^3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx}{b} + \int \frac{C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{(3B)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^3} + C\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{C\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b^2} + \frac{(3B)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{3b^3d\sqrt{b\sec(c+dx)}} + \frac{C\int \sqrt{b\sec(c+dx)}\,dx}{3b^4}$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{3b^3d\sqrt{b\sec(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^4}$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^4d} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{2C\sin(c+dx)}{3b^3d\sqrt{b\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\left(9BE\left(\frac{1}{2}(c+dx)\middle|2\right)+5C\,\text{EllipticF}\right.}{15b}$$

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] +
 5*C*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*C + 3*B*Cos[c + d*x]
)*Sin[c + d*x]))/(15*b^4*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.85

method	result
parts	$\frac{2B\left(3i\,\mathrm{EllipticE}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cdots\right.}{}$
default	$-\frac{6iB\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)}{5}+\frac{6iB\,\mathrm{EllipticE}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\cdots}{5}$

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/5*B/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^3*(3*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I
)*cos(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)+6*I*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)
),I)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(-cot(d*x+c)+csc(d*x+c)),I)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-3*I
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
ot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)+cos(d*x+c)^2*sin(d*x+c)+sin(d*x+c)*cos(
d*x+c)+3*sin(d*x+c))-2/3*C/d/(b*sec(d*x+c))^(1/2)/b^3*(I*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c
)),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{-5i\sqrt{2}C\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})\cdots}{}$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*C*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*C*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstra
```

```
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(b)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c)^2 + 5*C*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(
d*x + c))/(b^4*d)
```

Sympy [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\int \frac{(B+C\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(7/2),x)
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="
maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="
giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2),x)
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2), x)
```

3.53 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 176

$$\begin{aligned}\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx &= \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\ &+ \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^5d} \\ &+ \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{2C\sin(c+dx)}{5b^3d(b\sec(c+dx))^{3/2}} + \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}}\end{aligned}$$

```
[Out] 2/7*B*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(5/2)+2/5*C*sin(d*x+c)/b^3/d/(b*sec(d
*x+c))^(3/2)+10/21*B*sin(d*x+c)/b^4/d/(b*sec(d*x+c))^(1/2)+6/5*C*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
/b^4/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+10/21*B*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*(b*sec(d*x+c))^(1/2)/b^5/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4132, 3854, 3856, 2720, 12, 16, 2719}

$$\begin{aligned}\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx &= \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^5d} \\ &+ \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}} + \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\ &+ \frac{2C\sin(c+dx)}{5b^3d(b\sec(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}}\end{aligned}$$

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (6*C*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(21*b^5*d) + (2*B*Sin[c + d*x])/(7*b^2*d*(b*Sec[c + d*x])^(5/2)) + (
2*C*Sin[c + d*x])/(5*b^3*d*(b*Sec[c + d*x])^(3/2)) + (10*B*Sin[c + d*x])/(2
1*b^4*d*Sqrt[b*Sec[c + d*x]])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 16</u>

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

<u>Rule 2719</u>

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

<u>Rule 2720</u>

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

<u>Rule 3854</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

<u>Rule 3856</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

<u>Rule 4132</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx}{b} + \int \frac{C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{(5B)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b^3} + C\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}} + \frac{(5B)\int \sqrt{b\sec(c+dx)}\,dx}{21b^5} + \frac{C\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx}{b^2}$$
$$= \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{2C\sin(c+dx)}{5b^3d(b\sec(c+dx))^{3/2}} + \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}} + \frac{(3C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^4} + \frac{\left(5B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^5}$$
$$= \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^5d} + \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{2C\sin(c+dx)}{5b^3d(b\sec(c+dx))^{3/2}} + \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}} + \frac{(3C)\int \sqrt{\cos(c+dx)}\,dx}{5b^4\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^5d} + \frac{2B\sin(c+dx)}{7b^2d(b\sec(c+dx))^{5/2}} + \frac{2C\sin(c+dx)}{5b^3d(b\sec(c+dx))^{3/2}} + \frac{10B\sin(c+dx)}{21b^4d\sqrt{b\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(252C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(\right.}{}$$

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(9/2),x]
[Out] (Sqrt[b*Sec[c + d*x]]*(252*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*C*Cos[c +
d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*b^5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 582, normalized size of antiderivative = 3.31

method	result
parts	$-\frac{2B\left(5i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+5i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx}{21d\sqrt{b\sec(dx+c)}\,b^4}$
default	$-\frac{2\left(-63iC\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticE}(i(-\cot(dx+c)+\csc(dx+c)),i)\sec(dx+c)+50iB\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{Elli}}{}$

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/21*B/d/(b*sec(d*x+c))^(1/2)/b^4*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+5*I*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c
)+csc(d*x+c)),I)*sec(d*x+c)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))+2/5*C/d
/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^4*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d
*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)+6*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)-6*
I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-
cot(d*x+c)+csc(d*x+c)),I)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-3*I*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+
c)+csc(d*x+c)),I)*sec(d*x+c)+cos(d*x+c)^2*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)+
3*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \frac{-25i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{}$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="
fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 63*I*sqrt(2)*C*sqrt(b)*weierstrassZeta(-4, 0, weier
```

```
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*C*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(15*B*cos(d*x + c)^3 + 21*C*cos(d*x + c)^2 + 25*B*cos(d*x +
c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx=\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="
maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx=\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(9/2),x, algorithm="
giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx = \int \frac{\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

```
[In] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(9/2),x)
[Out] int((B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(9/2), x)
```

3.54 $\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 122

$$\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3B\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{(5A+4C)\tan(c+dx)}{5d}+\frac{3B\sec(c+dx)\tan(c+dx)}{8d}+\frac{B\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{C\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{(5A+4C)\tan^3(c+dx)}{15d}$$

```
[Out] 3/8*B*arctanh(sin(d*x+c))/d+1/5*(5*A+4*C)*tan(d*x+c)/d+3/8*B*sec(d*x+c)*tan
(d*x+c)/d+1/4*B*sec(d*x+c)^3*tan(d*x+c)/d+1/5*C*sec(d*x+c)^4*tan(d*x+c)/d+1
/15*(5*A+4*C)*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4132, 3853, 3855, 4131, 3852}

$$\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(5A+4C)\tan^3(c+dx)}{15d}+\frac{(5A+4C)\tan(c+dx)}{5d}+\frac{3B\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3B\tan(c+dx)\sec(c+dx)}{8d}+\frac{C\tan(c+dx)\sec^4(c+dx)}{5d}$$

```
[In] Int[Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]])/(8*d) + ((5*A + 4*C)*Tan[c + d*x])/(5*d) + (3*B
*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
```

```
(C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*A + 4*C)*Tan[c + d*x]^3)/(15*d
)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \sec^5(c+dx)\,dx + \int \sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{B\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} \\
&\quad + \frac{1}{4}(3B)\int \sec^3(c+dx)\,dx + \frac{1}{5}(5A+4C)\int \sec^4(c+dx)\,dx \\
&= \frac{3B\sec(c+dx)\tan(c+dx)}{8d} + \frac{B\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} \\
&\quad + \frac{1}{8}(3B)\int \sec(c+dx)\,dx - \frac{(5A+4C)\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{5d}
\end{aligned}
$$

$$= \frac{3B\text{arctanh}(\sin(c+dx))}{8d} + \frac{(5A+4C)\tan(c+dx)}{5d} + \frac{3B\sec(c+dx)\tan(c+dx)}{8d} + \frac{B\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{C\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(5A+4C)\tan^3(c+dx)}{15d}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

$$= \frac{45B\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(45B\sec(c+dx)+30B\sec^3(c+dx)+8(15(A+C)+5(A+2C\cdots\right.}{120d}$$

```
[In] Integrate[Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (45*B*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*B*Sec[c + d*x] + 30*B*Sec[c
+ d*x]^3 + 8*(15*(A + C) + 5*(A + 2*C)*Tan[c + d*x]^2 + 3*C*Tan[c + d*x]^4)
))/(120*d)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+B\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-C\left(-\frac{8}{15}\cdots\right.}{d}$
default	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+B\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-C\left(-\frac{8}{15}\cdots\right.}{d}$
parts	$-\frac{A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{B\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\cdots$
risch	$-\frac{i\left(45B\,e^{9i(dx+c)}+210B\,e^{7i(dx+c)}-240A\,e^{6i(dx+c)}-560A\,e^{4i(dx+c)}-640C\,e^{4i(dx+c)}-210B\,e^{3i(dx+c)}-400A\,e^{2i(dx+c)}\cdots\right.}{60d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4(25A+29C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{(8A-5B+8C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{(8A+5B+8C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(32A-3B+16C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}+\frac{(32A+\cdots}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^5}$
parallelrisch	$\frac{-450B\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+450B\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\cdots}{120d(\cos(5dx+5\cdots}$

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-C*(-8/15-1/5*sec(d*x+c)^4-
4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \sec^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$$
$$= \frac{45 \, B \cos\left(dx+c\right)^5 \log\left(\sin\left(dx+c\right)+1\right) - 45 \, B \cos\left(dx+c\right)^5 \log\left(-\sin\left(dx+c\right)+1\right) + 2 \, \left(16 \, \left(5 \, A+4 \, C\right) \cos\right.}{240 \, d \cos\left(dx\right.}$$

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/240*(45*B*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*B*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(16*(5*A + 4*C)*cos(d*x + c)^4 + 45*B*cos(d*x + c)^3
 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 30*B*cos(d*x + c) + 24*C)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$$
$$= \int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^4\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$$
$$= \frac{80 \, \left(\tan\left(dx+c\right)^3 + 3 \, \tan\left(dx+c\right)\right)A + 16 \, \left(3 \, \tan\left(dx+c\right)^5 + 10 \, \tan\left(dx+c\right)^3 + 15 \, \tan\left(dx+c\right)\right)C - 15 \, B}{240 \, d}$$

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A + 16*(3*tan(d*x + c)^5 + 10*t
an(d*x + c)^3 + 15*tan(d*x + c))*C - 15*B*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(110) = 220$.

Time = 0.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.02

$$\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{45\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/120*(45*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*B*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(120*A*tan(1/2*d*x + 1/2*c)^9 - 75*B*tan(1/2*d*x + 1/2*c)
^9 + 120*C*tan(1/2*d*x + 1/2*c)^9 - 320*A*tan(1/2*d*x + 1/2*c)^7 + 30*B*tan
(1/2*d*x + 1/2*c)^7 - 160*C*tan(1/2*d*x + 1/2*c)^7 + 400*A*tan(1/2*d*x + 1/
2*c)^5 + 464*C*tan(1/2*d*x + 1/2*c)^5 - 320*A*tan(1/2*d*x + 1/2*c)^3 - 30*B
*tan(1/2*d*x + 1/2*c)^3 - 160*C*tan(1/2*d*x + 1/2*c)^3 + 120*A*tan(1/2*d*x
+ 1/2*c) + 75*B*tan(1/2*d*x + 1/2*c) + 120*C*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.61

$$\int \sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\left(2\,A-\frac{5\,B}{4}+2\,C\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{B}{2}-\frac{16\,A}{3}-\frac{8\,C}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,A}{3}+\frac{116\,C}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}\cdots\right.\right.}$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x)^4,x)
```

```
[Out] (3*B*atanh(tan(c/2 + (d*x)/2)))/(4*d) - (tan(c/2 + (d*x)/2)^5*((20*A)/3 + (
116*C)/15) + tan(c/2 + (d*x)/2)*(2*A + (5*B)/4 + 2*C) + tan(c/2 + (d*x)/2)^
9*(2*A - (5*B)/4 + 2*C) - tan(c/2 + (d*x)/2)^3*((16*A)/3 + B/2 + (8*C)/3) -
 tan(c/2 + (d*x)/2)^7*((16*A)/3 - B/2 + (8*C)/3))/(d*(5*tan(c/2 + (d*x)/2)^
2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2
)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.55 $\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{(4A+3C)\text{arctanh}(\sin(c+dx))}{8d}+\frac{B\tan(c+dx)}{d}+\frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}$$

```
[Out] 1/8*(4*A+3*C)*arctanh(sin(d*x+c))/d+B*tan(d*x+c)/d+1/8*(4*A+3*C)*sec(d*x+c)
*tan(d*x+c)/d+1/4*C*sec(d*x+c)^3*tan(d*x+c)/d+1/3*B*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4132, 3852, 4131, 3853, 3855}

$$\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{(4A+3C)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}+\frac{C\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[In] Int[Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (B*Tan[c + d*x])/d + ((4*A + 3*
C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (B*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \sec^4(c+dx)\,dx+\int \sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\
&= \frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(4A+3C)\int \sec^3(c+dx)\,dx\\
&\quad-\frac{B\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{B\tan(c+dx)}{d}+\frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{B\tan^3(c+dx)}{3d}+\frac{1}{8}(4A+3C)\int \sec(c+dx)\,dx
\end{aligned}$$

$$= \frac{(4A+3C)\mathrm{arctanh}(\sin(c+dx))}{8d} + \frac{B\tan(c+dx)}{d} + \frac{(4A+3C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{C\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{B\tan^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3(4A+3C)\mathrm{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3(4A+3C)\sec(c+dx)+6C\sec^3(c+dx)+8B(3+\tan^2\right.}{24d}$$

```
[In] Integrate[Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (3*(4*A + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*A + 3*C)*Sec[c +
d*x] + 6*C*Sec[c + d*x]^3 + 8*B*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\mathrm{t}\right.}{d}$
default	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\mathrm{t}\right.}{d}$
parts	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{B\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+}{8}\right.\right.}{}$
parallelrisch	$\frac{-48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{3C}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(A+\frac{3C}{4}\right)\ln\left(\tan\right.}{24d(\cos(4dx+4c)+4\cos(2d}$
norman	$\frac{\frac{(4A+5C-8B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(4A+5C+8B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(12A-9C-40B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{(12A-9C+40B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}-\frac{(4A}{}$
risch	$-\frac{i\left(12A\,e^{7i(dx+c)}+9C\,e^{7i(dx+c)}+12A\,e^{5i(dx+c)}+33C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-12A\,e^{3i(dx+c)}-33C\,e^{3i(dx+c)}-64B\,e^{2i(\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*(-2/3-1/
3*sec(d*x+c)^2)*tan(d*x+c)+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{3\,(4\,A+3\,C)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,A+3\,C)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2}{48\,d\cos\left(dx+c\right)^4}$$

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/48*(3*(4*A + 3*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A + 3*C)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*B*cos(d*x + c)^3 + 3*(4*A + 3*C
)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*C)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.43

$$\int \sec^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)B-3\,C\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\mathrm{si}\right.\right.}{48\,d}$$

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B - 3*C*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(89) = 178.

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.37

$$\int \sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\,(4\,A+3\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,A+3\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*A + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A + 3*C)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*A*tan(1/2*d*x + 1/2*c)^7 - 24*B*tan(
1/2*d*x + 1/2*c)^7 + 15*C*tan(1/2*d*x + 1/2*c)^7 - 12*A*tan(1/2*d*x + 1/2*c
)^5 + 40*B*tan(1/2*d*x + 1/2*c)^5 + 9*C*tan(1/2*d*x + 1/2*c)^5 - 12*A*tan(1
/2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/2*c)^3 + 9*C*tan(1/2*d*x + 1/2*c)^
3 + 12*A*tan(1/2*d*x + 1/2*c) + 24*B*tan(1/2*d*x + 1/2*c) + 15*C*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+\frac{3\,C}{4}\right)}{d}$$
$$+\frac{\left(A-2\,B+\frac{5\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{10\,B}{3}-A+\frac{3\,C}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,C}{4}-\frac{10\,B}{3}-A\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(A\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(A + (3*C)/4))/d + (tan(c/2 + (d*x)/2)*(A + 2*B
+ (5*C)/4) + tan(c/2 + (d*x)/2)^7*(A - 2*B + (5*C)/4) - tan(c/2 + (d*x)/2)^
3*(A + (10*B)/3 - (3*C)/4) + tan(c/2 + (d*x)/2)^5*((10*B)/3 - A + (3*C)/4))
/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)
^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.56 $\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{B\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{(3A+2C)\tan(c+dx)}{3d}+\frac{B\sec(c+dx)\tan(c+dx)}{2d}+\frac{C\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] 1/2*B*arctanh(sin(d*x+c))/d+1/3*(3*A+2*C)*tan(d*x+c)/d+1/2*B*sec(d*x+c)*tan
(d*x+c)/d+1/3*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4132, 3853, 3855, 4131, 3852, 8}

$$\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(3A+2C)\tan(c+dx)}{3d}+\frac{B\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}+\frac{C\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[In] Int[Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*A + 2*C)*Tan[c + d*x])/(3*d) + (B*Sec
[c + d*x]*Tan[c + d*x])/(2*d) + (C*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \sec^3(c+dx)\,dx + \int \sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{B\sec(c+dx)\tan(c+dx)}{2d} + \frac{C\sec^2(c+dx)\tan(c+dx)}{3d} \\
&\quad + \frac{1}{2}B\int \sec(c+dx)\,dx + \frac{1}{3}(3A+2C)\int \sec^2(c+dx)\,dx \\
&= \frac{B\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{C\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{(3A+2C)\operatorname{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3d}
\end{aligned}
$$

$$= \frac{B\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{(3A+2C)\tan(c+dx)}{3d} + \frac{B\sec(c+dx)\tan(c+dx)}{2d} + \frac{C\sec^2(c+dx)\tan(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3B\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(6(A+C)+3B\sec(c+dx)+2C\tan^2(c+dx)\right)}{6d}$$

```
[In] Integrate[Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (3*B*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + C) + 3*B*Sec[c + d*x] + 2
*C*Tan[c + d*x]^2))/(6*d)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{A\tan(dx+c)+B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{A\tan(dx+c)+B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{A\tan(dx+c)}{d}+\frac{B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
norman	$\frac{\frac{4(3A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{(2A-B+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{(2A+B+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{B\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$
risch	$-\frac{i\left(3B\,e^{5i(dx+c)}-6A\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-6A-4C\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{B\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{B\ln(}{}$
parallelrisch	$\frac{-9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(6A+4C)\sin}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*tan(d*x+c)+B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
))-C*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \sec^2(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$= \frac{3\,B\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,B\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,\left(3\,A+2\,C\right)\cos\left(d\right.\right.}{12\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
[Out] 1/12*(3*B*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*B*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(2*(3*A + 2*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*C)
*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$= \int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \sec^2(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$= \frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)C-3\,B\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+}{12\,d}$$

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C - 3*B*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*A*tan(d*x
 + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(70) = 140.

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C}{}}{6\,d}$$

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/6*(3*B*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*B*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 6*
C*tan(1/2*d*x + 1/2*c)^5 - 12*A*tan(1/2*d*x + 1/2*c)^3 - 4*C*tan(1/2*d*x +
1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{(2\,A-B+2\,C)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,A-\frac{4\,C}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(2\,A+B+2\,C)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x)^2,x)
```

```
[Out] (B*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(2*A + B + 2*C) - tan
(c/2 + (d*x)/2)^3*(4*A + (4*C)/3) + tan(c/2 + (d*x)/2)^5*(2*A - B + 2*C))/(
d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 -
 1))
```

3.57 $\int \sec(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 51

$$\int \sec(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(2*A+C)*arctanh(sin(d*x+c))/d+B*tan(d*x+c)/d+1/2*C*sec(d*x+c)*tan(d*x+c
)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4132, 3852, 8, 4131, 3855}

$$\int \sec(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)}{d}+\frac{C\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c + d*x])/d + (C*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{C\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}(2A+C)\int \sec(c+dx)\,dx - \frac{B\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{(2A+C)\text{arctanh}(\sin(c+dx))}{2d} + \frac{B\tan(c+dx)}{d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$
\begin{aligned}
&\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx \\
&= \frac{A\text{arctanh}(\sin(c+dx))}{d} + \frac{C\text{arctanh}(\sin(c+dx))}{2d} \\
&\quad + \frac{B\tan(c+dx)}{d} + \frac{C\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

```
[In] Integrate[Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Tan[c +
d*x])/d + (C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+B\tan(dx+c)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+B\tan(dx+c)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{B\tan(dx+c)}{d}+\frac{C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{-\left(A+\frac{C}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(A+\frac{C}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+B\sin(2dx+2c)+C\sin}{d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{(2B+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2B-C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}-\frac{(2A+C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2A+C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{i\left(C\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-C\,e^{i(dx+c)}-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)A}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)C}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)A}{d}+\ln($

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*ln(sec(d*x+c)+tan(d*x+c))+B*tan(d*x+c)+C*(1/2*sec(d*x+c)*tan(d*x+c)+
1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

$$\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(2\,A+C)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,A+C)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(2\,B\cos}{4\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/4*((2*A + C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A + C)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(2*B*cos(d*x + c) + C)*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(A+B\sec{(c+dx)}+C\sec^2{(c+dx)}\right)\sec{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx =$$
$$-\frac{C\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,A\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4\,d}$$

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] -1/4*(C*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*A*log(sec(d*x + c) + tan(d*x + c)) - 4*B*tan(d*x + c
))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{(2\,A+C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,A+C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-C\tan\left(\ldots\right.\right.}{\left(\ldots\right.}}{2\,d}$$

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*((2*A + C)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A + C)*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) - 2*(2*B*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x + 1/2
*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.75

$$\int \sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,A+C\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,B-C\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,B+C\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*A + C))/d - (tan(c/2 + (d*x)/2)^3*(2*B - C) -
 tan(c/2 + (d*x)/2)*(2*B + C))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)
/2)^2 + 1))
```

3.58 $\int (A+B\sec(c+dx)+C\sec^2(c+dx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=Ax+\frac{B\text{arctanh}(\sin(c+dx))}{d}+\frac{C\tan(c+dx)}{d}$$

```
[Out] A*x+B*arctanh(sin(d*x+c))/d+C*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3855, 3852, 8}

$$\int \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=Ax+\frac{B\text{arctanh}(\sin(c+dx))}{d}+\frac{C\tan(c+dx)}{d}$$

```
[In] Int[A + B*Sec[c + d*x] + C*Sec[c + d*x]^2,x]
[Out] A*x + (B*ArcTanh[Sin[c + d*x]])/d + (C*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= Ax + B\int \sec(c+dx)\,dx + C\int \sec^2(c+dx)\,dx\\ &= Ax + \frac{B\text{arctanh}(\sin(c+dx))}{d} - \frac{C\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= Ax + \frac{B\text{arctanh}(\sin(c+dx))}{d} + \frac{C\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)\,dx = Ax + \frac{B\text{arctanh}(\sin(c+dx))}{d} + \frac{C\tan(c+dx)}{d}$$

```
[In] Integrate[A + B*Sec[c + d*x] + C*Sec[c + d*x]^2,x]
[Out] A*x + (B*ArcTanh[Sin[c + d*x]])/d + (C*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

method	result	size
default	$Ax + \frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{C\tan(dx+c)}{d}$	35
parts	$Ax + \frac{B\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{C\tan(dx+c)}{d}$	35
derivativedivides	$\frac{(dx+c)A+B\ln(\sec(dx+c)+\tan(dx+c))+C\tan(dx+c)}{d}$	37
risch	$Ax - \frac{B\ln(e^{i(dx+c)}-i)}{d} + \frac{B\ln(e^{i(dx+c)}+i)}{d} + \frac{2iC}{d(e^{2i(dx+c)}+1)}$	62
parallelrisch	$\frac{-B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+C\sin(dx+c)}{d\cos(dx+c)} + Ax$	67
norman	$\frac{Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - Ax - \frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1} + \frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	87

```
[In] int(A+B*sec(d*x+c)+C*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] A*x+B/d*ln(sec(d*x+c)+tan(d*x+c))+C*tan(d*x+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\,dx = \frac{2\,Adx\cos(dx + c) + B\cos(dx + c)\log(\sin(dx + c) + 1) - B\cos(dx + c)\log(-\sin(dx + c) + 1) + 2\,C\sin(dx + c)}{2\,d\cos(dx + c)}$$

```
[In] integrate(A+B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="fricas")
[Out] 1/2*(2*A*d*x*cos(d*x + c) + B*cos(d*x + c)*log(sin(d*x + c) + 1) - B*cos(d*
x + c)*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\,dx = \int \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\,dx$$

```
[In] integrate(A+B*sec(d*x+c)+C*sec(d*x+c)**2,x)
[Out] Integral(A + B*sec(c + d*x) + C*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\,dx = Ax + \frac{B\log(\sec(dx + c) + \tan(dx + c))}{d} + \frac{C\tan(dx + c)}{d}$$

```
[In] integrate(A+B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="maxima")
[Out] A*x + B*log(sec(d*x + c) + tan(d*x + c))/d + C*tan(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = Ax+\frac{B\left(\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)\right)}{4d}+\frac{C\tan(dx+c)}{d}$$

```
[In] integrate(A+B*sec(d*x+c)+C*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] A*x + 1/4*B*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d + C*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.96

$$\int \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2A\,\mathrm{atan}\left(\frac{64A^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64A^3+64AB^2}+\frac{64AB^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64A^3+64AB^2}\right)}{d}+\frac{2B\,\mathrm{atanh}\left(\frac{64B^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64A^2B+64B^3}+\frac{64A^2B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64A^2B+64B^3}\right)}{d}-\frac{2C\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

```
[In] int(A + B/cos(c + d*x) + C/cos(c + d*x)^2,x)
```

```
[Out] (2*A*atan((64*A^3*tan(c/2 + (d*x)/2))/(64*A*B^2 + 64*A^3) + (64*A*B^2*tan(c
/2 + (d*x)/2))/(64*A*B^2 + 64*A^3)))/d + (2*B*atanh((64*B^3*tan(c/2 + (d*x)
/2))/(64*A^2*B + 64*B^3) + (64*A^2*B*tan(c/2 + (d*x)/2))/(64*A^2*B + 64*B^3
)))/d - (2*C*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.59 $\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = Bx+\frac{C\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{A\sin(c+dx)}{d}$$

```
[Out] B*x+C*arctanh(sin(d*x+c))/d+A*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4132, 8, 4130, 3855}

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\sin(c+dx)}{d}+\frac{C\operatorname{arctanh}(\sin(c+dx))}{d}+Bx$$

```
[In] Int[Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] B*x + (C*ArcTanh[Sin[c + d*x]])/d + (A*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int 1\,dx+\int \cos(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\
&= Bx+\frac{A\sin(c+dx)}{d}+C\int \sec(c+dx)\,dx\\
&= Bx+\frac{C\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{A\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\
&= Bx+\frac{C\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{A\cos(dx)\sin(c)}{d}+\frac{A\cos(c)\sin(dx)}{d}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] B*x + (C*ArcTanh[Sin[c + d*x]])/d + (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d
*x])/d
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

method	result	siz
derivativedivides	$\frac{A\sin(dx+c)+B(dx+c)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$	37
default	$\frac{A\sin(dx+c)+B(dx+c)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$	37
parallelrisch	$\frac{Bxd+A\sin(dx+c)-C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	47
risch	$Bx-\frac{iA\,e^{i(dx+c)}}{2d}+\frac{iA\,e^{-i(dx+c)}}{2d}+\frac{\ln(e^{i(dx+c)}+i)C}{d}-\frac{\ln(e^{i(dx+c)}-i)C}{d}$	74
norman	$\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-Bx-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}+\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{C\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	11

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*sin(d*x+c)+B*(d*x+c)+C*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,Bdx+C\log\left(\sin\left(dx+c\right)+1\right)-C\log\left(-\sin\left(dx+c\right)+1\right)+2\,A\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(2*B*d*x + C*log(sin(d*x + c) + 1) - C*log(-sin(d*x + c) + 1) + 2*A*sin
(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,(dx+c)B+C(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,A\sin(dx+c)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(2*(d*x + c)*B + C*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*
A*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(dx+c)B+C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*B + C*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 2*A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.52

$$\int \cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,C\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*C*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*sin(c + d*x))/d
```

3.60 $\int \cos^2(c+dx) \left(A + B \sec(c + dx) + C \sec^2(c + dx)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 42

$$\int \cos^2(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{1}{2}(A+2C)x+\frac{B\sin(c+dx)}{d}+\frac{A\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(A+2*C)*x+B*sin(d*x+c)/d+1/2*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4132, 2717, 4130, 8}

$$\int \cos^2(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{A\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(A+2C)+\frac{B\sin(c+dx)}{d}$$

```
[In] Int[Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] ((A + 2*C)*x)/2 + (B*Sin[c + d*x])/d + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \cos(c+dx)\,dx + \int \cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{B\sin(c+dx)}{d} + \frac{A\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(A+2C)\int 1\,dx \\
&= \frac{1}{2}(A+2C)x + \frac{B\sin(c+dx)}{d} + \frac{A\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$
\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = Cx + \frac{A(c+dx)}{2d} + \frac{B\cos(dx)\sin(c)}{d} + \frac{B\cos(c)\sin(dx)}{d} + \frac{A\sin(2(c+dx))}{4d}
$$

```
[In] Integrate[Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] C*x + (A*(c + d*x))/(2*d) + (B*Cos[d*x]*Sin[c])/d + (B*Cos[c]*Sin[d*x])/d +
 (A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
risch	$\frac{Ax}{2}+Cx+\frac{B\sin(dx+c)}{d}+\frac{A\sin(2dx+2c)}{4d}$
parallelrisch	$\frac{A\sin(2dx+2c)+4B\sin(dx+c)+2(A+2C)xd}{4d}$
derivativedivides	$\frac{A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\sin(dx+c)+C(dx+c)}{d}$
default	$\frac{A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\sin(dx+c)+C(dx+c)}{d}$
norman	$\frac{\left(-\frac{A}{2}-C\right)x+\left(-\frac{A}{2}-C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{A}{2}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\left(\frac{A}{2}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d}-\frac{(A-2B)}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*A*x+C*x+B*sin(d*x+c)/d+1/4*A/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(A+2\,C)dx+(A\cos(dx+c)+2\,B)\sin(dx+c)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
[Out] 1/2*((A + 2*C)*d*x + (A*cos(d*x + c) + 2*B)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int (A+B\sec(c+dx)+C\sec^2(c+dx))\cos^2(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))A+4\,(dx+c)C+4\,B\sin\left(dx+c\right)}{4\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A + 4*(d*x + c)*C + 4*B*sin(d*x + c))
/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

$$\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{(dx+c)(A+2\,C)-\frac{2\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*((d*x + c)*(A + 2*C) - 2*(A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x +
1/2*c)^3 - A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\,x}{2}+C\,x+\frac{A\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] (A*x)/2 + C*x + (A*sin(2*c + 2*d*x))/(4*d) + (B*sin(c + d*x))/d
```

3.61 $\int \cos^3(c+dx) \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 56

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{Bx}{2}+\frac{(A+C)\sin(c+dx)}{d}+\frac{B\cos(c+dx)\sin(c+dx)}{2d}-\frac{A\sin^3(c+dx)}{3d}$$

```
[Out] 1/2*B*x+(A+C)*sin(d*x+c)/d+1/2*B*cos(d*x+c)*sin(d*x+c)/d-1/3*A*sin(d*x+c)^3
/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4132, 2715, 8, 4129, 3092}

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{(A+C)\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

```
[In] Int[Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (B*x)/2 + ((A + C)*Sin[c + d*x])/d + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d) -
(A*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^2(c+dx)\,dx + \int \cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\
&= \frac{B\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}B\int 1\,dx + \int \cos(c+dx)\left(C+A\cos^2(c+dx)\right)\,dx\\
&= \frac{Bx}{2} + \frac{B\cos(c+dx)\sin(c+dx)}{2d} - \frac{\text{Subst}\left(\int \left(A+C-Ax^2\right)\,dx, x, -\sin(c+dx)\right)}{d}\\
&= \frac{Bx}{2} + \frac{(A+C)\sin(c+dx)}{d} + \frac{B\cos(c+dx)\sin(c+dx)}{2d} - \frac{A\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{6Bc+6Bdx+3(3A+4C)\sin(c+dx)+3B\sin(2(c+dx))+A\sin(3(c+dx))}{12d}$$

```
[In] Integrate[Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
[Out] (6*B*c + 6*B*d*x + 3*(3*A + 4*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + A*Si
n[3*(c + d*x)])/(12*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{3B\sin(2dx+2c)+A\sin(3dx+3c)+(9A+12C)\sin(dx+c)+6Bxd}{12d}$
derivativedivides	$\frac{\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C\sin(dx+c)}{d}$
default	$\frac{\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+B\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+C\sin(dx+c)}{d}$
risch	$\frac{Bx}{2}+\frac{3A\sin(dx+c)}{4d}+\frac{C\sin(dx+c)}{d}+\frac{A\sin(3dx+3c)}{12d}+\frac{B\sin(2dx+2c)}{4d}$
norman	$\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{(2A-B+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{Bx}{2}-Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{2}-\frac{(2A-3B-6C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}-\frac{(2}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*(3*B*sin(2*d*x+2*c)+A*sin(3*d*x+3*c)+(9*A+12*C)*sin(d*x+c)+6*B*x*d)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,Bdx+\left(2\,A\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)+4\,A+6\,C\right)\sin\left(dx+c\right)}{6\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*B*d*x + (2*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 4*A + 6*C)*sin(d*x
+ c))/d
```

Sympy [F]

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\cos^3\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = -\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)A-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)B-12\,C\sin\left(dx+c\right)}{12\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*B - 12*C*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(50) = 100.

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.46

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\left(dx+c\right)B+\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,A\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*B + 2*(6*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*
c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 + 4*A*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1
/2*d*x + 1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c) + 6
*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \cos^3(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) \, dx = \frac{B\,x}{2}+\frac{2\,A\,\sin\left(c+d\,x\right)}{3\,d}+\frac{C\,\sin\left(c+d\,x\right)}{d}+\frac{B\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{A\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (B*x)/2 + (2*A*sin(c + d*x))/(3*d) + (C*sin(c + d*x))/d + (B*cos(c + d*x)*s
in(c + d*x))/(2*d) + (A*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.62 $\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 88

$$\begin{aligned}\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx \\ = \frac{1}{8}(3A+4C)x+\frac{B\sin(c+dx)}{d}+\frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} \\ +\frac{A\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{B\sin^3(c+dx)}{3d}\end{aligned}$$

```
[Out] 1/8*(3*A+4*C)*x+B*sin(d*x+c)/d+1/8*(3*A+4*C)*cos(d*x+c)*sin(d*x+c)/d+1/4*A*
cos(d*x+c)^3*sin(d*x+c)/d-1/3*B*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4132, 2713, 4130, 2715, 8}

$$\begin{aligned}\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx \\ = \frac{(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{A\sin(c+dx)\cos^3(c+dx)}{4d} \\ +\frac{1}{8}x(3A+4C)-\frac{B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}\end{aligned}$$

```
[In] Int[Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((3*A + 4*C)*x)/8 + (B*Sin[c + d*x])/d + ((3*A + 4*C)*Cos[c + d*x]*Sin[c +
d*x])/(8*d) + (A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (B*Sin[c + d*x]^3)/(3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \cos^3(c+dx)\,dx + \int \cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{A\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3A+4C)\int \cos^2(c+dx)\,dx \\
&\quad - \frac{B\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d} \\
&= \frac{B\sin(c+dx)}{d} + \frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad + \frac{A\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{B\sin^3(c+dx)}{3d} + \frac{1}{8}(3A+4C)\int 1\,dx
\end{aligned}
$$

$$= \frac{1}{8}(3A+4C)x + \frac{B\sin(c+dx)}{d} + \frac{(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{B\sin^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{36Ac+48cC+36Adx+48Cdx+96B\sin(c+dx)-32B\sin^3(c+dx)+24(A+C)\sin(2(c+dx))+3A\text{ s}}{96d}$$

[In] Integrate[Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]

[Out] (36*A*c + 48*c*C + 36*A*d*x + 48*C*d*x + 96*B*Sin[c + d*x] - 32*B*Sin[c + d
x]^3 + 24(A + C)*Sin[2*(c + d*x)] + 3*A*Sin[4*(c + d*x)])/(96*d)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{24(A+C)\sin(2dx+2c)+8B\sin(3dx+3c)+3A\sin(4dx+4c)+72B\sin(dx+c)+36d\left(A+\frac{4C}{3}\right)x}{96d}$
risch	$\frac{3Ax}{8}+\frac{Cx}{2}+\frac{3B\sin(dx+c)}{4d}+\frac{A\sin(4dx+4c)}{32d}+\frac{B\sin(3dx+3c)}{12d}+\frac{A\sin(2dx+2c)}{4d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+C\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\left(-\frac{3A}{8}-\frac{C}{2}\right)x+\left(-\frac{9A}{8}-\frac{3C}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{3A}{4}-C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{3A}{4}+C\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3A}{8}+\frac{C}{2}\right)x\tan\left(\frac{dx}{2}\right.$

[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/96*(24*(A+C)*sin(2*d*x+2*c)+8*B*sin(3*d*x+3*c)+3*A*sin(4*d*x+4*c)+72*B*si
n(d*x+c)+36*d*(A+4/3*C)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$=\frac{3\,(3\,A+4\,C)dx+\left(6\,A\cos\left(dx+c\right)^3+8\,B\cos\left(dx+c\right)^2+3\,(3\,A+4\,C)\cos\left(dx+c\right)+16\,B\right)\sin\left(dx+c\right)}{24\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/24*(3*(3*A + 4*C)*d*x + (6*A*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(3*A
 + 4*C)*cos(d*x + c) + 16*B)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$=\int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\cos^4\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \cos^4(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\, dx$$
$$=\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))A-32\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)B+24\,(2\,dx}{96\,d}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*B + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.27

$$\int \cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\,(dx+c)(3\,A+4\,C) - \frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*(d*x + c)*(3*A + 4*C) - 2*(15*A*tan(1/2*d*x + 1/2*c)^7 - 24*B*tan(1
/2*d*x + 1/2*c)^7 + 12*C*tan(1/2*d*x + 1/2*c)^7 - 9*A*tan(1/2*d*x + 1/2*c)^
5 - 40*B*tan(1/2*d*x + 1/2*c)^5 + 12*C*tan(1/2*d*x + 1/2*c)^5 + 9*A*tan(1/2
*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^3
 - 15*A*tan(1/2*d*x + 1/2*c) - 24*B*tan(1/2*d*x + 1/2*c) - 12*C*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{3\,A\,x}{8} + \frac{C\,x}{2} + \frac{A\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{A\,\sin\left(4\,c+4\,d\,x\right)}{32\,d} + \frac{B\,\sin\left(3\,c+3\,d\,x\right)}{12\,d} + \frac{C\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{3\,B\,\sin\left(c+d\,x\right)}{4\,d}$$

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (3*A*x)/8 + (C*x)/2 + (A*sin(2*c + 2*d*x))/(4*d) + (A*sin(4*c + 4*d*x))/(32
*d) + (B*sin(3*c + 3*d*x))/(12*d) + (C*sin(2*c + 2*d*x))/(4*d) + (3*B*sin(c
 + d*x))/(4*d)
```

3.63 $\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 98

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{3Bx}{8}+\frac{(A+C)\sin(c+dx)}{d}+\frac{3B\cos(c+dx)\sin(c+dx)}{8d}+\frac{B\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{(2A+C)\sin^3(c+dx)}{3d}+\frac{A\sin^5(c+dx)}{5d}$$

```
[Out] 3/8*B*x+(A+C)*sin(d*x+c)/d+3/8*B*cos(d*x+c)*sin(d*x+c)/d+1/4*B*cos(d*x+c)^3
*sin(d*x+c)/d-1/3*(2*A+C)*sin(d*x+c)^3/d+1/5*A*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4132, 2715, 8, 4129, 3092, 380}

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = -\frac{(2A+C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{A\sin^5(c+dx)}{5d}+\frac{B\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3B\sin(c+dx)\cos(c+dx)}{8d}+\frac{3Bx}{8}$$

```
[In] Int[Cos[c + d*x]^5*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*B*x)/8 + ((A + C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - ((2*A + C)*Sin[c + d*x]^3)/(3*d
) + (A*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= B\int \cos^4(c+dx)\,dx + \int \cos^5(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\ &= \frac{B\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3B)\int \cos^2(c+dx)\,dx\\ &\quad + \int \cos^3(c+dx)\left(C+A\cos^2(c+dx)\right)\,dx\end{aligned}$$

$$= \frac{3B\cos(c+dx)\sin(c+dx)}{8d} + \frac{B\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3B)\int 1\,dx - \frac{\text{Subst}\left(\int (1-x^2)\left(A+C-Ax^2\right)\,dx, x, -\sin(c+dx)\right)}{d}$$

$$= \frac{3Bx}{8} + \frac{3B\cos(c+dx)\sin(c+dx)}{8d} + \frac{B\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(2A+C)x^2+Ax^4\right)\,dx, x, -\sin(c+dx)\right)}{d}$$

$$= \frac{3Bx}{8} + \frac{(A+C)\sin(c+dx)}{d} + \frac{3B\cos(c+dx)\sin(c+dx)}{8d} + \frac{B\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{(2A+C)\sin^3(c+dx)}{3d} + \frac{A\sin^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{180Bc+180Bdx+60(5A+6C)\sin(c+dx)+120B\sin(2(c+dx))+50A\sin(3(c+dx))+40C\sin(3(c}{480d}$$

```
[In] Integrate[Cos[c + d*x]^5*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (180*B*c + 180*B*d*x + 60*(5*A + 6*C)*Sin[c + d*x] + 120*B*Sin[2*(c + d*x)]
 + 50*A*Sin[3*(c + d*x)] + 40*C*Sin[3*(c + d*x)] + 15*B*Sin[4*(c + d*x)] +
6*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{(50A+40C)\sin(3dx+3c)+120B\sin(2dx+2c)+15B\sin(4dx+4c)+6A\sin(5dx+5c)+(300A+360C)\sin(dx+c)+180Bxd}{480d}$
derivativedivides	$\frac{\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+B\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+}{3}}{d}$
default	$\frac{\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+B\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+}{3}}{d}$
risch	$\frac{3Bx}{8}+\frac{5A\sin(dx+c)}{8d}+\frac{3C\sin(dx+c)}{4d}+\frac{A\sin(5dx+5c)}{80d}+\frac{B\sin(4dx+4c)}{32d}+\frac{5A\sin(3dx+3c)}{48d}+\frac{\sin(3dx+3c)C}{12d}$
norman	$\frac{-\frac{3Bx}{8}-\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{15Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8}+\frac{15Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{2}+\frac{3Bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8}-\frac{(8A-9B+40C}{1}}{}$

[In] int(cos(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/480*((50*A+40*C)*sin(3*d*x+3*c)+120*B*sin(2*d*x+2*c)+15*B*sin(4*d*x+4*c)+6*A*sin(5*d*x+5*c)+(300*A+360*C)*sin(d*x+c)+180*B*x*d)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{45\,Bdx+\left(24\,A\cos\left(dx+c\right)^4+30\,B\cos\left(dx+c\right)^3+8\,(4\,A+5\,C)\cos\left(dx+c\right)^2+45\,B\cos\left(dx+c\right)+64\,A}{120\,d}$$

[In] integrate(cos(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")

[Out] 1/120*(45*B*d*x + (24*A*cos(d*x + c)^4 + 30*B*cos(d*x + c)^3 + 8*(4*A + 5*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 64*A + 80*C)*sin(d*x + c))/d

Sympy [F]

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\cos^5\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**5*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{32\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)A+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\right.\right.}{480\,d}$$

```
[In] integrate(cos(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B - 160*(sin(d*x + c
)^3 - 3*sin(d*x + c))*C)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(88) = 176.

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.27

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{45\left(dx+c\right)B+\frac{2\left(120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/120*(45*(d*x + c)*B + 2*(120*A*tan(1/2*d*x + 1/2*c)^9 - 75*B*tan(1/2*d*x
+ 1/2*c)^9 + 120*C*tan(1/2*d*x + 1/2*c)^9 + 160*A*tan(1/2*d*x + 1/2*c)^7 -
```

```
30*B*tan(1/2*d*x + 1/2*c)^7 + 320*C*tan(1/2*d*x + 1/2*c)^7 + 464*A*tan(1/2*
d*x + 1/2*c)^5 + 400*C*tan(1/2*d*x + 1/2*c)^5 + 160*A*tan(1/2*d*x + 1/2*c)^
3 + 30*B*tan(1/2*d*x + 1/2*c)^3 + 320*C*tan(1/2*d*x + 1/2*c)^3 + 120*A*tan(
1/2*d*x + 1/2*c) + 75*B*tan(1/2*d*x + 1/2*c) + 120*C*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{3\,B\,x}{8}+\frac{5\,A\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{A\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{C\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{5\,A\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,C\,\sin\left(c+d\,x\right)}{4\,d}$$

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (3*B*x)/8 + (5*A*sin(3*c + 3*d*x))/(48*d) + (A*sin(5*c + 5*d*x))/(80*d) + (
B*sin(2*c + 2*d*x))/(4*d) + (B*sin(4*c + 4*d*x))/(32*d) + (C*sin(3*c + 3*d*
x))/(12*d) + (5*A*sin(c + d*x))/(8*d) + (3*C*sin(c + d*x))/(4*d)
```

3.64 $\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 132

$$\begin{aligned}&\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\&=\frac{1}{16}(5A+6C)x+\frac{B\sin(c+dx)}{d}+\frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d}\\&\quad+\frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d}\\&\quad+\frac{A\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}\end{aligned}$$

```
[Out] 1/16*(5*A+6*C)*x+B*sin(d*x+c)/d+1/16*(5*A+6*C)*cos(d*x+c)*sin(d*x+c)/d+1/24
*(5*A+6*C)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*A*cos(d*x+c)^5*sin(d*x+c)/d-2/3*B*
sin(d*x+c)^3/d+1/5*B*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4132, 2713, 4130, 2715, 8}

$$\begin{aligned}&\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx\\&=\frac{(5A+6C)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5A+6C)\sin(c+dx)\cos(c+dx)}{16d}\\&\quad+\frac{A\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{1}{16}x(5A+6C)\\&\quad+\frac{B\sin^5(c+dx)}{5d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin(c+dx)}{d}\end{aligned}$$

```
[In] Int[Cos[c + d*x]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((5*A + 6*C)*x)/16 + (B*Sin[c + d*x])/d + ((5*A + 6*C)*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) + ((5*A + 6*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (A*Cos[c
 + d*x]^5*Sin[c + d*x])/(6*d) - (2*B*Sin[c + d*x]^3)/(3*d) + (B*Sin[c + d*x
]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \cos^5(c+dx)\,dx + \int \cos^6(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx\\
&= \frac{A\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5A+6C)\int \cos^4(c+dx)\,dx\\
&\quad - \frac{B\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{B\sin(c+dx)}{d}+\frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{A\cos^5(c+dx)\sin(c+dx)}{6d} \\
&\quad-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}+\frac{1}{8}(5A+6C)\int\cos^2(c+dx)\,dx \\
&=\frac{B\sin(c+dx)}{d}+\frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad+\frac{A\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}+\frac{1}{16}(5A+6C)\int 1\,dx \\
&=\frac{1}{16}(5A+6C)x+\frac{B\sin(c+dx)}{d}+\frac{(5A+6C)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{(5A+6C)\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad+\frac{A\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{2B\sin^3(c+dx)}{3d}+\frac{B\sin^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$
\int\cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx
$$
$$
=\frac{960B\sin(c+dx)-640B\sin^3(c+dx)+192B\sin^5(c+dx)+5(60Ac+72cC+60Adx+72Cdx+(45A}{960d}
$$

```
[In] Integrate[Cos[c + d*x]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (960*B*Sin[c + d*x] - 640*B*Sin[c + d*x]^3 + 192*B*Sin[c + d*x]^5 + 5*(60*A
*c + 72*c*C + 60*A*d*x + 72*C*d*x + (45*A + 48*C)*Sin[2*(c + d*x)] + (9*A +
 6*C)*Sin[4*(c + d*x)] + A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(225A+240C)\sin(2dx+2c)+(45A+30C)\sin(4dx+4c)+100B\sin(3dx+3c)+12B\sin(5dx+5c)+5A\sin(6dx+6c)+600B\sin}{960d}$
derivativedivides	$\frac{A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+C\left(\frac{(\cos(}{}\right.}{d}$
default	$\frac{A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{B\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+C\left(\frac{(\cos(}{}\right.}{d}$
risch	$\frac{5Ax}{16}+\frac{3Cx}{8}+\frac{5B\sin(dx+c)}{8d}+\frac{A\sin(6dx+6c)}{192d}+\frac{B\sin(5dx+5c)}{80d}+\frac{3A\sin(4dx+4c)}{64d}+\frac{\sin(4dx+4c)C}{32d}+\frac{5B\sin}{}$
norman	$\frac{\left(-\frac{5A}{16}-\frac{3C}{8}\right)x+\left(-\frac{45A}{16}-\frac{27C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{25A}{16}-\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{25A}{16}-\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{5A}{16}+\frac{3}{}\right.}{}$

```
[In] int(cos(d*x+c)^6*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*((225*A+240*C)*sin(2*d*x+2*c)+(45*A+30*C)*sin(4*d*x+4*c)+100*B*sin(3*
d*x+3*c)+12*B*sin(5*d*x+5*c)+5*A*sin(6*d*x+6*c)+600*B*sin(d*x+c)+300*d*x*(A
+6/5*C))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{15\,(5\,A+6\,C)dx+\left(40\,A\cos{(dx+c)}^5+48\,B\cos{(dx+c)}^4+10\,(5\,A+6\,C)\cos{(dx+c)}^3+64\,B\cos{(dx+}\right.}{240\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas
")
```

```
[Out] 1/240*(15*(5*A + 6*C)*d*x + (40*A*cos(d*x + c)^5 + 48*B*cos(d*x + c)^4 + 10
*(5*A + 6*C)*cos(d*x + c)^3 + 64*B*cos(d*x + c)^2 + 15*(5*A + 6*C)*cos(d*x
+ c) + 128*B)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.87

$$\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=$$
$$-\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)A-64\left(3\sin\left(dx+c\right)^5-\right.}{960\,d}$$

```
[In] integrate(cos(d*x+c)^6*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima
")
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x
+ c))*B - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.15

$$\int \cos^6(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{15\,(dx+c)(5\,A+6\,C)-\frac{2\left(165\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-240\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+150\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-560\,B\,\mathrm{ta}\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^6*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/240*(15*(d*x + c)*(5*A + 6*C) - 2*(165*A*tan(1/2*d*x + 1/2*c)^11 - 240*B*
tan(1/2*d*x + 1/2*c)^11 + 150*C*tan(1/2*d*x + 1/2*c)^11 - 25*A*tan(1/2*d*x
+ 1/2*c)^9 - 560*B*tan(1/2*d*x + 1/2*c)^9 + 210*C*tan(1/2*d*x + 1/2*c)^9 +
450*A*tan(1/2*d*x + 1/2*c)^7 - 1248*B*tan(1/2*d*x + 1/2*c)^7 + 60*C*tan(1/2
```

```
*d*x + 1/2*c)^7 - 450*A*tan(1/2*d*x + 1/2*c)^5 - 1248*B*tan(1/2*d*x + 1/2*c
)^5 - 60*C*tan(1/2*d*x + 1/2*c)^5 + 25*A*tan(1/2*d*x + 1/2*c)^3 - 560*B*tan
(1/2*d*x + 1/2*c)^3 - 210*C*tan(1/2*d*x + 1/2*c)^3 - 165*A*tan(1/2*d*x + 1/
2*c) - 240*B*tan(1/2*d*x + 1/2*c) - 150*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \cos^6(c+dx) \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{5\,A\,x}{16}+\frac{3\,C\,x}{8}+\frac{15\,A\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,A\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{A\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{5\,B\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{B\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{C\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{C\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{5\,B\,\sin\left(c+d\,x\right)}{8\,d}$$

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
```

```
[Out] (5*A*x)/16 + (3*C*x)/8 + (15*A*sin(2*c + 2*d*x))/(64*d) + (3*A*sin(4*c + 4*
d*x))/(64*d) + (A*sin(6*c + 6*d*x))/(192*d) + (5*B*sin(3*c + 3*d*x))/(48*d)
 + (B*sin(5*c + 5*d*x))/(80*d) + (C*sin(2*c + 2*d*x))/(4*d) + (C*sin(4*c +
4*d*x))/(32*d) + (5*B*sin(c + d*x))/(8*d)
```

3.65 $\int (b\sec(c+dx))^{3/2} \left(A+B\sec(c+dx)+C\sec^2(c+dx\right.$

Optimal result

Integrand size = 33, antiderivative size = 178

$$\int (b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = -\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] 2/3*B*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d-2/5*b^2*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(
d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b*(5*A+3*C)*sin(d*x+c)*(b*sec(d*x+c))
^(1/2)/d+2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/5*C*(
b*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4132, 3853, 3856, 2720, 4131, 2719}

$$\int (b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = -\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b(5A+3C)\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2C\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}$$

```
[In] Int[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*b^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[b*Sec[c + d*x]])/(3*d) + (2*b*(5*A + 3*C)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*B*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*C*(b*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\int (b\sec(c+dx))^{5/2}\,dx}{b} + \int (b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} \\
&\quad + \frac{1}{3}(bB)\int \sqrt{b\sec(c+dx)}\,dx + \frac{1}{5}(5A+3C)\int (b\sec(c+dx))^{3/2}\,dx \\
&= \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} - \frac{1}{5}\left(b^2(5A+3C)\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&\quad + \frac{1}{3}\left(bB\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} - \frac{\left(b^2(5A+3C)\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}
\end{aligned}$$

$$= -\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b(5A+3C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2C(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.71 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.73

$$\int (b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2e^{-ic}(-1+e^{2ic})\cos^3(c+dx)\csc(c)\left(5B-15Ae^{i(c+dx)}-3Ce^{i(c+dx)}-30Ae^{3i(c+dx)}-2\right.}{}$$

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(-1 + E^((2*I)*c))*Cos[c + d*x]^3*Csc[c]*(5*B - 15*A*E^(I*(c + d*x)) - 3
*C*E^(I*(c + d*x)) - 30*A*E^((3*I)*(c + d*x)) - 24*C*E^((3*I)*(c + d*x)) -
5*B*E^((4*I)*(c + d*x)) - 15*A*E^((5*I)*(c + d*x)) - 9*C*E^((5*I)*(c + d*x)
) - (5*I)*B*(1 + E^((2*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2] + (5*A + 3*C)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(b*Sec[c + d*x])^(3/2)
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*c)*(1 + E^((2*I)*(c +
d*x)))^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 949, normalized size of antiderivative = 5.33

method	result	size
parts	Expression too large to display	949
default	Expression too large to display	998

```
[In] int((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*A/d*(I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-I*EllipticE(I*(cot(d*x+c)-csc
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(
d*x+c)^2+2*I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*EllipticE(I*(cot(d*x+c)-
csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)+I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-sin(d*x+c))*(b*
sec(d*x+c))^(1/2)*b/(cos(d*x+c)+1)+2/3*B/d*(b*sec(d*x+c))^(1/2)*b*(-I*Ellip
ticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+tan(d*x+c))+2/5*C/d
*(b*sec(d*x+c))^(1/2)*b/(cos(d*x+c)+1)*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+
c)^2-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)^2+6*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*cos
(d*x+c)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*cos(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-3*
I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+3*sin(d*x+c)+tan(d*x+c)+sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.26

$$\int (b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{-5i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5}{}$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 3*C)*
b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(5*A + 3*C)*b^(3/2)*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*(5*A + 3*C)*b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*C*b
)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (b\sec(c+dx))^{3/2} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \int (b\sec(c+dx))^{\frac{3}{2}} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$$

```
[In] integrate((b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
[Out] Integral((b*sec(c + d*x))**(3/2)*(A + B*sec(c + d*x) + C*sec(c + d*x)**2),
x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{3/2} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^{\frac{3}{2}} dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x
)
```

Giac [F]

$$\int (b\sec(c+dx))^{3/2} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^{\frac{3}{2}} dx$$

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(A+\frac{B}{\cos\left(c+d\,x\right)}+\frac{C}{{\cos\left(c+d\,x\right)}^2}\right)\,dx$$

```
[In] int((b/cos(c + d*x))^(3/2)*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] int((b/cos(c + d*x))^(3/2)*(A + B/cos(c + d*x) + C/cos(c + d*x)^2), x)
```

3.66 $\int \sqrt{b\sec(c+dx)}(A+B\sec(c+dx)+C\sec^2(c+dx))$

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \sqrt{b\sec(c+dx)}(A+B\sec(c+dx)+C\sec^2(c+dx))\,dx = -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] -2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*B*sin(d*x+c)*(b
*sec(d*x+c))^(1/2)/d+2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))
^(1/2)/d+2/3*C*(b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{4132, 3853, 3856, 2719, 4131, 2720\}$

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{d} - \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2C\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}$$

```
[In] Int[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*b*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*d) + (2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*C*Sqrt[b*
Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
```

```
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\int (b\sec(c+dx))^{3/2}\,dx}{b} + \int \sqrt{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx \\
&= \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} \\
&\quad - (bB)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx + \frac{1}{3}(3A+C)\int \sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d} \\
&\quad - \frac{(bB)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&\quad + \frac{1}{3}\left((3A+C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&\quad + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} \\
&\quad + \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2C\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.22

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

$$= \frac{4\sqrt{b\sec(c+dx)}(A+B\sec(c+dx)+C\sec^2(c+dx))\left(-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3B\left(1+e^{2i(c+dx)}\right)+3B\left(-1+e^{2ic}\right)\sqrt{\right.}\right.}{}}{3d(A}$$

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-I)*Sqrt
[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*B*(1 + E^((2*I)*(c +
 d*x))) + 3*B*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*A + C)*E^(I*(c + d*x))*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5
/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec
[c + d*x]]*(3*B*Cos[d*x]*Csc[c] + C*Tan[c + d*x])))/(3*d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 611, normalized size of antiderivative = 4.49

method	result
parts	$-\frac{2 i A (\cos(dx+c)+1) \sqrt{\frac{1}{\cos(dx+c)+1}}\, \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)), i) \sqrt{b \sec(dx+c)}}{d} + \frac{2B\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{}$
default	Expression too large to display

```
[In] int((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*I*A/d*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(b*sec(d*x+c))^(1/2)+2*B/d*
(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)^2-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c
)^2+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(
d*x+c)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(-cot(d*x+c)+csc(d*x+c)),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)+sin(d*x+c))*(b*s
ec(d*x+c))^(1/2)/(cos(d*x+c)+1)-2/3*C/d*(b*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+cs
c(d*x+c)),I)*cos(d*x+c)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\cos{(dx+c)}\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}(3i\,A+i\,C)}{}$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*cos(d*x + c
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B
*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
2*(3*B*cos(d*x + c) + C)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
)
```

Sympy [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \sqrt{b\sec{(c+dx)}}\left(A+B\sec{(c+dx)}+C\sec^2{(c+dx)}\right)\,dx$$

```
[In] integrate((b*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2), x)
```

Maxima [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(C\sec{(dx+c)}^2+B\sec{(dx+c)}+A\right)\sqrt{b\sec{(dx+c)}}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$
$$=\int \sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}+\frac{C}{{\cos\left(c+d\,x\right)}^2}\right)\,dx$$

```
[In] int((b/cos(c + d*x))^(1/2)*(A + B/cos(c + d*x) + C/cos(c + d*x)^2),x)
[Out] int((b/cos(c + d*x))^(1/2)*(A + B/cos(c + d*x) + C/cos(c + d*x)^2), x)
```

3.67 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b/d+2*C*tan(d*x+c)/d/(b*sec(d*x+c))^(1
/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4132, 3856, 2720, 4131, 2719}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d
*x]])/(b*d) + (2*C*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int\sqrt{b\sec(c+dx)}\,dx}{b}+\int\frac{A+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}+(A-C)\int\frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&\quad+\frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}
\end{aligned}
$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}} + \frac{(A-C)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$= \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd} + \frac{2C\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

$$= \frac{2e^{-idx}\sqrt{b\sec(c+dx)}(-i\cos(dx)+\sin(dx))\left(-3A\cos(c+dx)+3C\cos(c+dx)+3iB\sqrt{\cos(c+dx)}\,\text{Ellip}\right.}{}$$

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*((-I)*Cos[d*x] + Sin[d*x])*(-3*A*Cos[c + d*x] + 3*C
*Cos[c + d*x] + (3*I)*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A -
 C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*C*Sin[c + d*x]))/(3*b*d*E^(I*d*x))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 847, normalized size of antiderivative = 7.70

method	result	size
parts	Expression too large to display	847
default	Expression too large to display	945

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2*A/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*EllipticE(I*(-cot(d*x+c)+csc(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)-I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
```

```
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot
(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(
d*x+c)),I)*sec(d*x+c)+sin(d*x+c))-2*I*B/d*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-cot(d*x+c)+csc(d*x+c)),I)/(b*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)-2*C/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*EllipticE(I*(-cot(
d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)-I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+
c)),I)-2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-I*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot
(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0}{}$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + sqrt(2)*(I*A - I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*A + I*C)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))) + 2*C*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\int \frac{A+B\sec{(c+dx)}+C\sec^2{(c+dx)}}{\sqrt{b\sec{(c+dx)}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\int \frac{C\sec{(dx+c)}^2+B\sec{(dx+c)}+A}{\sqrt{b\sec{(dx+c)}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\int \frac{C\sec{(dx+c)}^2+B\sec{(dx+c)}+A}{\sqrt{b\sec{(dx+c)}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\int \frac{A+\frac{B}{\cos(c+d\,x)}+\frac{C}{{\cos(c+d\,x)}^2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

3.68 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/3*(A+3*C)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d+2/3*A*tan(d*x+c)/d/(b*sec(
d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4132, 3856, 2719, 4130, 2720}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+d}}{3b^2d}+\frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c
 + d*x]])/(3*b^2*d) + (2*A*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}+\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx\\
&= \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}+\frac{(A+3C)\int \sqrt{b\sec(c+dx)}\,dx}{3b^2}+\frac{B\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\\
&\quad+\frac{\left((A+3C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2}
\end{aligned}
$$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2A\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(6iB\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-2i(A+3C\right.}{3b}$$

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*((6*I)*B*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - (2*I)
*(A + 3*C)*E^(I*(c + d*x))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*((-3*I)*B + A*Sin[c + d*x])))/(3*b*d
*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 608, normalized size of antiderivative = 5.20

method	result
parts	$-\frac{2A\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx\right.}{3d\sqrt{b\sec(dx+c)}\,b}$
default	Expression too large to display

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*A/d/(b*sec(d*x+c))^(1/2)/b*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d
*x+c)),I)*sec(d*x+c)-sin(d*x+c))+2*B/b/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2
)*(I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*EllipticF(I*(-cot(d*x+c)+csc(d*x+
c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
```

```
)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(
I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I*(1/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*
x+c)),I)*sec(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)+sin(d*x+c))-2*I*C/
d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)/b/(b*sec
(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(b\sec(c + dx))^{3/2}}\,dx = \frac{2\,A\sqrt{\frac{b}{\cos(dx+c)}}\cos{(dx+c)}\sin{(dx+c)} + \sqrt{2}(-i\,A - 3i\,C)\sqrt{b}\mathrm{we}$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(2*A*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-I*A - 3
*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + s
qrt(2)*(I*A + 3*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*B*sqrt(b)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/
(b^2*d)
```

Sympy [F]

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(b\sec(c + dx))^{3/2}}\,dx = \int \frac{A + B\sec{(c + dx)} + C\sec^2{(c + dx)}}{(b\sec{(c + dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(3/2),
x)
```

Maxima [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x
)
```

Giac [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\int \frac{A+\frac{B}{\cos(c+d\,x)}+\frac{C}{{\cos(c+d\,x)}^2}}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

3.69 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 150

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[Out] 2/3*B*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)+2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/
cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*
sec(d*x+c))^(1/2)/b^3/d+2/5*A*tan(d*x+c)/d/(b*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4132, 3854, 3856, 2720, 4130, 2719}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[
b*Sec[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b
*Sec[c + d*x]])/(3*b^3*d) + (2*B*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]
]) + (2*A*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b} + \int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}} + \frac{B\int \sqrt{b\sec(c+dx)}\,dx}{3b^3} + \frac{(3A+5C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}$$

$$= \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}} + \frac{(3A+5C)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3}$$

$$= \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{e^{-idx}\sqrt{b\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(10B\sqrt{\cos(c+dx)}\,\text{Ell}\right.}{}$$

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(10*B*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] - (2*I)*(3*A + 5*C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c
 + d*x]*((6*I)*(3*A + 5*C) + 10*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)])))/(1
5*b^3*d*E^(I*d*x))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 956, normalized size of antiderivative = 6.37

method	result	size
parts	Expression too large to display	956
default	Expression too large to display	1004

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*A/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^2*(3*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I
)*cos(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)+6*I*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)
),I)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(I*(-cot(d*x+c)+csc(d*x+c)),I)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-3*I
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
ot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)+cos(d*x+c)^2*sin(d*x+c)+sin(d*x+c)*cos(
d*x+c)+3*sin(d*x+c))-2/3*B/d/(b*sec(d*x+c))^(1/2)/b^2*(I*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c
)),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-sin(d*x+c))+2*C/b^2/d/(cos(d*x+c
)+1)/(b*sec(d*x+c))^(1/2)*(I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*EllipticF
(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+
c)),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d
*x+c)+sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.17

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-5i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{}$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(3*I*A + 5*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*A*cos(d*x + c)^2 + 5*B*cos(d*x + c))*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(5/2),
x)
```

Maxima [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\sec(dx+c)^2+B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x
)
```

Giac [F]

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\int \frac{A+\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

3.70 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 185

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[Out] 2/5*B*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(3/2)+2/21*(5*A+7*C)*sin(d*x+c)/b^3/d
/(b*sec(d*x+c))^(1/2)+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c)
)^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^4
/d+2/7*A*tan(d*x+c)/d/(b*sec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4132, 3854, 3856, 2719, 4130, 2720}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}$$

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b
*Sec[c + d*x]])/(21*b^4*d) + (2*B*Sin[c + d*x])/(5*b^2*d*(b*Sec[c + d*x])^(
3/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*Sqrt[b*Sec[c + d*x]]) + (2*A
*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx}{b}+\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx\\
&= \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\\
&\quad+\frac{(3B)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^3}+\frac{(5A+7C)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b^2}\\
&= \frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\\
&\quad+\frac{(5A+7C)\int \sqrt{b\sec(c+dx)}\,dx}{21b^4}+\frac{(3B)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}\\
&\quad+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}+\frac{\left((5A+7C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^4}\\
&= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&\quad+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d}\\
&\quad+\frac{2B\sin(c+dx)}{5b^2d(b\sec(c+dx))^{3/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2A\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$
\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx = \frac{504iB\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-40i(5A+7C
$$

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((504*I)*B*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - (40*I)
*(5*A + 7*C)*E^(I*(c + d*x))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
+ d*x))] + Sqrt[1 + E^((2*I)*(c + d*x))]*(5*(23*A + 28*C)*Sin[c + d*x] + 3*
((-84*I)*B + 14*B*Sin[2*(c + d*x)] + 5*A*Sin[3*(c + d*x)])))/(210*b^3*d*Sqr
t[1 + E^((2*I)*(c + d*x))]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.92

method	result
parts	$-\frac{2A\left(5i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+5i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(-\cot(\right.}{21d\sqrt{b\sec(dx+c)}\,b^3}$
default	Expression too large to display

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*A/d/(b*sec(d*x+c))^(1/2)/b^3*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+5*I*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c
)+csc(d*x+c)),I)*sec(d*x+c)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))+2/5*B/d
/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^3*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d
*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(I*(-cot(d*x+c)+csc(d*x+c)),I)*cos(d*x+c)+6*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)-6*
I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-
cot(d*x+c)+csc(d*x+c)),I)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-3*I*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+
c)+csc(d*x+c)),I)*sec(d*x+c)+cos(d*x+c)^2*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)+
3*sin(d*x+c))-2/3*C/d/(b*sec(d*x+c))^(1/2)/b^3*(I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+I
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
ot(d*x+c)+csc(d*x+c)),I)*sec(d*x+c)-sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,A+7i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-5i\,A-7i\,C)\sqrt{b}\text{we}}{}$$

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm
="fricas")
```

[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*A*cos(d*x + c)^3 + 21*B*cos(d*x + c)^2 + 5*(5*A + 7*C)*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^4*d)

Sympy [F]

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(b\sec(c + dx))^{7/2}}\,dx = \int \frac{A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)}{\left(b\sec\left(c + dx\right)\right)^{\frac{7}{2}}}\,dx$$

[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(7/2),x)

[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(7/2), x)

Maxima [F]

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(b\sec(c + dx))^{7/2}}\,dx = \int \frac{C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{7}{2}}}\,dx$$

[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(7/2), x)

Giac [F]

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(b\sec(c + dx))^{7/2}}\,dx = \int \frac{C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A}{\left(b\sec\left(dx + c\right)\right)^{\frac{7}{2}}}\,dx$$

[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx=\int \frac{A+\frac{B}{\cos(c+d\,x)}+\frac{C}{\cos(c+d\,x)^2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2),x)
[Out] int((A + B/cos(c + d*x) + C/cos(c + d*x)^2)/(b/cos(c + d*x))^(7/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
         fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
         gamma,loggamma,digamma,zeta,polylog,LambertW,
         elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```